A MULTI-ATTRIBUTE COMBINATORIAL AUCTION APPROACH TO ELECTRONIC PROCUREMENT MECHANISM DESIGN

Jian Chen
Professor and Director, Research Center for Contemporary Management,
Tsinghua University, Beijing 100084 China
jchen@mail.tsinghua.edu.cn

He Huang
Doctoral Program
School of Economics and Management
Tsinghua University, Beijing 100084 China
huangh02@mails.tsinghua.edu.cn

Robert J. Kauffman (contact author)
Professor and Director, MIS Research Center
Chair, Information and Decision Science Department
Carlson School of Management, University of Minnesota
Minneapolis, MN 55455
rkauffman@csom.umn.edu

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ABSTRACT

This article focuses on mechanism design in public procurement settings involving combinatorial auctions. An important difference between combinatorial and forward auctions for procurement is that multiple attributes of the items for sale must be represented, since buying is more complicated than selling. We propose a new mechanism for multi-attribute combinatorial procurement auctions by revising mechanisms associated with the past proposals of Groves and Clark that are specifically designed to deal with the issue of social welfare maximization in the public procurement setting. Our multi-attribute Vickrey-Groves-Clarke (MA-VCG) mechanism is incentive-compatible, provides constraints on partial participation, is budget-balanced, and also is efficient in quasi-linear preferences. In consideration of the profit perspective of the auctioneer, we also propose a payment function to implement the goal of achieving minimal costs for the government auctioneer, and appropriate benefits for participating suppliers. We also provide a brief numerical illustration of our MA-VCG mechanism in action, as well as consideration of its limitations.

Keywords: Auctions, auction economics, combinatorial auctions, electronic auctions, government procurement, mechanism design, multi-attributes, optimal auctions, public procurement, social welfare.

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INTRODUCTION

Combinatorial auctions have been applied in a variety of environments involving economic transactions, and they have the potential to play an important role in electronic transactions for supply chain management procurement (Milgrom, 2004) and other contexts (Klemperer, 2004; Krishna, 2002). For example, Logistics.com in the transportation and shipping industry has conducted industry-specific B2B procurement combinatorial auctions. Other examples are: Net Exchange (www.nex.com), which procures transportation services for Sears Logistics (Ledyard, et al., 2002); Home Depot, which uses combinatorial auctions to procure trucking services (Elmaghraby and Keskinocak, 2003); and IBM, which does procurement through combinatorial auctions on behalf of Mars Incorporated (Hohner, et al., 2003). In many application settings, the auctioneers in combinatorial auctions for supplies procurement are not only business enterprises, they also include governments and other public sector organizations that are likely to benefit (Krishna, 2002). In addition, combinatorial auctions can be used to procure goods as well as services or projects. In this paper, we will focus on the reverse auction version of combinatorial auctions for public or governmental procurement, where the emphasis is on social welfare.

An important aspect of combinatorial auctions is the efficacy of their mechanism design that supports the completion of transactions. In mechanism design for combinatorial auctions, it is important to consider the market microstructure, which typically involves a set of action rules for the entry of bids, as well as payment rules to determine the price or prices at which exchanges occur. Taken together, these characterize the market mechanism associated with a specific, real world combinatorial auction. For the purposes of this research, we define market mechanism as the “rules” of a combinatorial auction game, which define the actions available to agents and the method that is used to compute the outcome based on those actions, in line with the standard definitions used in the economics literature, such as Mas-Colell, Whinston and Green (1995), Jehle and Reny (2001) and Krishna (2002).

Examples of action rules include ascending and descending bidding, as well as sealed-bid auction rules. In an ascending price auction, the price is raised by bidders until only one bidder remains with the highest bid. In a descending price auction, the auctioneer starts the auction with a high price, and then lowers the price gradually. The first bidder who accepts the current price is the winner. In a sealed-bid auction, each
bidders submit a bid without knowing others’ bids, and the highest bidder is the winner. Usually, the payment rules involve a choice among a first-price, a second-price or a Vickrey-Clarke-Groves (VCG) payment scheme (Vickrey, 1961 and 1962; Clarke, 1971; Groves, 1973). A first-price auction involves a payment scheme in which the winner pays what she bids for the item or bundle. A second-price auction means that the winner pays what the second highest bidder bid for the sale item. The VCG payment scheme requires the winner to pay what she bids for the item or bundle, but the mechanism will refund to each winner the increased total value or surplus caused by her participation (Ausubel and Milgrom, 2002; Pekec and Rothkopf, 2003).

Combinatorial auctions have two features that greatly affect their design: their computational efficiency for determining the winner, and the economic efficiency of their mechanism (Jehiel and Moldovanu, 2003; Klemperer, 2004). Some special issues and problems occur in the context of combinatorial auctions (Klemperer, 2002). These include the exposure problem and bidding expressiveness (Nisan and Segal, 2002), the threshold problem and the moving efficiency problem (Cybernomics Inc., 2000; Banks, et al., 2001), and the ties problem in iterative combinatorial auctions (Pekec and Rothkopf, 2000). These issues have all been studied extensively. The interested reader should see the following surveys, which provide useful interpretive information on the definitions of the problems, the extent to which they diminish the value of combinatorial auction mechanisms, and the variety of approaches that have been suggested to permit their resolution: Englebrecht-Wiggans, Shubik and Stark, 1983; Klemperer (1999, 2003 and 2004), Krishna (2002), Pekec and Rothkopf (2003) and deVries and Vohra (2000). Another well known issue is called the winner determination problem, which has been explored in many different research papers to date (e.g., Rothkopf, Pekec and Harstad, 1998; Sandholm, 2000; Bikhchandani and Ostroy, 2002; Ausubel, and Milgrom, 2002). Combinatorial auctions are challenging, since it is difficult to specify exact solutions for combinatorial auction problems beyond a relatively small size.

From the perspective of economics and management science, however, the most pressing issues that exist in the optimal design of combinatorial auctions are allocation efficiency and revenue maximization (or cost minimization) (Jehiel and Moldovanu, 2003). Allocation efficiency occurs when the total value to the winners of the items being auctioned is maximized, and is a primary goal in auctions involving a
government auctioneer, such as the United States Federal Communications Commission for broadcast spectrum and transmission rights in radio networks, or airport commissions for landing and takeoff slots at a regional airport. In our context, we view allocative efficiency in terms of the minimization of the total costs of procurement, from the point of view of the government auctioneer/procurement agent. When the auctioneer is able to obtain the cheapest quality-satisfactory bundle of procurement goods, the auction’s outcome will be allocatively efficient. Allocative efficiency ensures social welfare maximization. Revenue maximization is also important from the perspective of the suppliers, who wish to maximize total revenue, and minimize their total cost (Pekec and Rothkopf, 2003).

However, these goals may contradict each other in different economic settings. For example, this can occur when there is a reserve price that does not permit the auction allocation priority ranking to be in synch with the ordered values of the bids that are made (Myerson, 1981). Krishna and Perry (2000) suggest that a tradeoff is probably necessary in most auction mechanism design cases. Mechanism design is intended to solve the problem of effective auction implementation. It presumes that there are self-interested rational agents with private information who act as participating suppliers in an auction. Their private information characterizes their supplier types and auction bidding demeanors. Some typical supplier descriptors include supplier preferences, the schedule of supplier values for different quantities, the time in the auction (e.g., early or late or both) when the supplier makes a bid, and so on.

The purpose of this paper is to design a mechanism for combinatorial reverse auction-based procurement of supplies to achieve maximum social welfare, which has the utilitarian social goal of optimizing total surplus for the auctioneer and the winning bidders. We emphasize that the role of the auctioneer that we wish to model in this work is a government or public entity procurement agent. For clarity related to the procurement in supply chain management, we will refer to bidders as suppliers. The specific characteristics of combinatorial auctions that we will treat in the procurement context involve heterogeneous goods involving multiple attributes. The property of heterogeneous items is the same property that we see in normal combinatorial auctions. However, the second property—multiple attributes—is somewhat unique in procurement auctions. Discussions about the application of the VCG mechanism in normal combinatorial auctions can be found in Ausubel and Milgrom (2002), Bikhchandani,
et al. (2002) and Parkes (2001), and elsewhere. The VCG mechanism will play a key role in the paper, providing a useful basis for our efforts to design a mechanism for combinatorial procurement auctions that addresses the key issues in social value maximization from a public procurement perspective.

To handle the multi-attribute combinatorial assignment problem in our model, we will revise the classical VCG mechanism by incorporating suppliers’ cost functions, and by depicting the multi-attribute nature of procurement combinatorial auctions. (The reader should recognize that the supplier’s cost function can be simultaneously used to characterize the auction bid she makes, and her non-profit willingness-to-pay or valuation for providing supplies in a given combinatorial auction.) We will show the allocative efficiency, the incentive compatibility in the sense of the appropriate dominant equilibrium strategies, the individual rationality of suppliers, and the budget-balancing capability of our mechanism. (For additional background on the variety of equilibrium concepts that have been discussed relative to the strategic analysis of auctions, the interested reader should see Wilson (2002).) We also will provide a uniqueness proof for the efficacy of the functional form of the payment function in our mechanism. We also are motivated to develop a means for the auctioneer to achieve greater expected revenue. To do this, we propose and evaluate another social choice function by revising the payoff function. Finally, we will show that the maximal expected revenue property holds for the revised mechanism by applying the revenue equivalence theorem (e.g., Vickrey, 1961 and 1962; Riley and Samuelson, 1981; Klemperer, 1999, 2003 and 2004) to achieve a Bayesian-Nash equilibrium (Krishna, 2000). For an innovative experimental test of the revenue equivalence theorem, see Tanjim and Morgan (2003). (Appendix A includes definitions of terms associated with combinatorial auctions and the MA-VCG analysis used throughout this paper.)

A MULTI-ATTRIBUTE VICKREY-CLARKE-GROVES (MA-VCG) MECHANISM

Suppose there are $n$ suppliers who make bids. A supplier is able to supply just one combination of goods in a combinatorial auction to one auctioneer. The bidding in the auction occurs around the prices at which the suppliers are willing to sell a bundle of procurement goods to an auctioneer. The auctioneer wants to procure one unit for each of $m$ different goods from suppliers. Let $X$ denote the set of all $m$ individual goods in a procurement auction that are to be delivered by the suppliers. We further define $B$ as the set of possible bidding combinations, with $B \subseteq X$, representing a combination of goods in the
procurement supplies set. The bidding language of supplier $j$ is given by $B_j = \{c_{jk}(.)\} \forall k \in B$. Here $c_{jk}(.)$ is supplier $j$’s cost function of supply for item $k$, an element of $B$, the set of possible bidding combinations. (See Appendix B for definitions of these and all other key modeling notation used in this paper.)

Based on the revelation principle for dominant strategies (Mas-Colell, Whinston and Green, 1995), we will design a direct revelation mechanism for the multi-attribute combinatorial procurement auction. Direct revelation in our setting means that the bids of any supplier will contain only two kinds of information: the bidding combination and the supplier’s cost function. Our MA-VCG mechanism can be represented in terms of three primary elements as the set MA-VCG: $\{A, \Theta, P\}$. $A$ is the set of all feasible allocations for the goods procured by the auctioneer from its suppliers with no assignment overlaps or omissions and no contract fulfillment failures. $\Theta$ denotes the quality standards for all goods. And, finally, $P$ denotes the payment function that identifies what winning suppliers will be paid. Under the MA-VCG mechanism, the suppliers are the bidders, and their types correspond to different cost functions that they face. In one specific allocation outcome, the suppliers obtain different assignments, which determine by whom and at what quality levels the relevant procurement goods are supplied.

The problem to be solved achieves the social goal of maximizing social surplus, and is specified as follows for one feasible allocation, $a$, across suppliers according to the allocation function:

$$\begin{align*}
\max_{\Theta, a, A \in A} \sum_{j=1}^{n} [v_j(\tilde{\theta}_{a^{-1}(j)}) - \hat{c}_j(\tilde{\theta}_{a^{-1}(j)})] \quad \text{(The Allocation Function)}
\end{align*}$$

where

$$v_j(\tilde{\theta}_{a^{-1}(j)}) = \sum_{k \in a^{-1}(j)} v_k(\theta_k)$$

$$\hat{c}_j(\tilde{\theta}_{a^{-1}(j)}) = \sum_{k \in a^{-1}(j)} \hat{c}_{jk}(\theta_k)$$

In the above expressions, $a$ denotes a specific optimal allocation outcome which is an element of $A$, the set of all feasible allocations by the auctioneer. In addition, $\Theta$ is the minimum quality requirement or quality threshold for all goods or items that are procured, and $a^{-1}(j)$ is the allocation to supplier $j$. The expression $\tilde{\theta}_{a^{-1}(j)}$ stands for the quality requirements for $n$ items allocated to supplier $j$, and it is a vector in $R^n$. 

Meanwhile, $\theta_k$ is the quality standard for procurement item $k$. Here, $v_k(\theta_k)$ represents the utility function for item $k$ with quality $\theta_k$, and supplier $j$’s bidding cost function, $\hat{c}_j(\cdot)$, for item $k$.

The MA-VCG mechanism optimizes the allocation of supplies to suppliers on the basis of Pareto efficiency, so that no supplier will be better off with any adjustment that is made to the allocation without another supplier being worse off. A related goal is maximizing the social surplus within the constraints of the applicable budget balance, an assumption that does not permit external subsidies. If the MA-VCG mechanism provides Pareto efficiency, then the allocation of supplies to suppliers must satisfy Equation 1’s allocation function.

Any given supplier $j$ who becomes one of the winners of the combinatorial auction will have an associated payment function, $P$, as follows:

$$P_j(\hat{c}_j) = V(J) + \hat{c}_j(\tilde{\theta}_{a^{-j}(j)}) - V(J \setminus j) \quad \text{(The Payment Function)}$$

where

$$V(J) = \max_{\theta_j, \theta_a, \theta_c} \sum_{j=1}^{n} [v_j(\tilde{\theta}_{a^{-j}(j)}) - \hat{c}_j(\tilde{\theta}_{a^{-j}(j)})]$$

$$V(J \setminus j) = \max_{\theta_i, \theta_a, \theta_c} \sum_{i \in [J \setminus j]} [v_i(\tilde{\theta}_{a^{-i}(i)}) - \hat{c}_i(\tilde{\theta}_{a^{-i}(i)})]$$

Let us consider the individual terms of Equation 4 in detail. The first term, $V(J)$, depicted in Equation 5 in its full form, is the maximum social surplus (based on the aggregation of all individual-level supplier utilities, $v_j$) that can be achieved with all bidding suppliers’ participation in the auction. The second term, as before, is the supplier’s reported cost associated with providing a winning set of goods to the auctioneer. The third term, shown in full form in Equation 6, is the maximum social surplus that can be achieved in the auction without supplier $j$’s participation. So supplier $j$’s payment is composed of the difference between the maximum achievable social surplus with or without her participation, plus the reported cost of her delivery of a winning combination of goods to the auctioneer that meet the quality requirement. Another way to interpret the payment function is to regard this payment as a transfer in value between auction agents. In particular, the transfer in our model is measured by the externality of supplier $j$ plus the utility of the combination of goods she supplies to auctioneer.
The surplus maximization problem in Equation 1 and the payment function in Equation 4 together represent the social choice function for our MA-VCG mechanism. Generally, a social choice function is a correspondence between supplier type to the allocation outcomes of an auction, which must satisfy some social goals. The implicit constraints embedded in Equations 5 and 6 for the optimization problems are all of the feasible allocations, which are partitions of all the procured goods. In the winner determination problem for combinatorial auctions, the feasibility and complexity of these allocations are important. However, it is common to assume that there is a mechanism that can solve the optimization problem to select the best outcome for the suppliers and the auctioneer. In our model, we assume that there are enough suppliers to participate in the procurement auction, so that an allocation outcome, in terms of the subsets of all goods that are procured, will always exist. (And, thus, we will put the associated computational problems aside for the moment.)

The Gibbard-Satterthwaite impossibility theorem (Gibbard, 1973; Satterthwaite, 1975) tells us that, for a very general class of problems, there is no hope of implementing satisfactory social choice functions in dominant strategies. Given this “negative” theorem, if we are to have any hope of implementing a desirable social choice function, based on the allocation function and the payment function in our model, we must either weaken the demands of our implementation or we must focus on more restricted environments. In this article, we will follow the latter course, pursuing the possibilities for implementing desirable social choice functions in dominant strategies when preferences take on a specific functional form. We will assume that the utility $u_j$ of any supplier $j$ has a quasi-linear form:

$$u_j = P_j(\theta_j) - \sum_{k \in a^{-1}(j)} c_{jk}(\theta_k) \quad \text{(The Quasi-Linear Utility Function)} \quad (7)$$

where $c_{jk}(.)$ once again is supplier $j$’s true cost function of item $k$.

**INCENTIVE COMPATIBILITY AND THE MA-VCG MECHANISM**

Next, we will show that the social choice function in our MA-VCG mechanism can be truthfully-implemented based on dominant strategies. For the purpose of maximizing her own utility, supplier $j$ will report her true cost function to the auctioneer as $\hat{c}_{jk}(.) = c_{jk}(.), \forall k \in a^{-1}(j)$. This results in our first proposition.
Proposition 1 (The Supplier’s Truth-Telling Strategy). Under the MA-VCG mechanism for combinatorial procurement auctions, truth-telling is a weakly-dominant strategy for any supplier.

Proof: Let $\Theta^*$ denote the optimal quality requirements for all goods which satisfy the allocation in Equation 1. If supplier $j$ is one of the winners, then her utility function can be expressed in terms of the quality requirements and the payment function in Equation 4 as:

$$u_j(\Theta^*, P_j) = u_j(\tilde{\Theta}^*_{a^{-1}(j)}, P_j)$$

$$= -\sum_{k=a^{-1}(j)} c_k(\tilde{\Theta}^*_k) + V(J) + \sum_{k=a^{-1}(j)} \hat{c}_k(\tilde{\Theta}^*_k) - V(J \setminus j)$$

$$= \sum_{i=1,i\neq j}^n [v_i(\tilde{\Theta}^*_{a^i(j)}) - \hat{c}_i(\tilde{\Theta}^*_{a^i(j)})] + [v_j(\tilde{\Theta}^*_{a^j(j)}) - c_j(\tilde{\Theta}^*_{a^j(j)})] - V(J \setminus j)$$

In above equality, the last term, $V(J \setminus j)$, is not influenced by supplier $j$’s revelation of her cost function information, since the auction’s utility is established in her absence. However, revelation of the true cost function indirectly affects the first two terms. Supplier $j$’s bid, $\hat{c}_k(\cdot), k \in a^{-1}(j)$, will affect her utility in Equation 8 based on the effect of the quality level for the supplies, $\Theta^*$, that is chosen based on payment function in Equation 4 earlier. The reader should also note the difference between the supplier’s cost functions as stated in Equation 4, $c_j(\tilde{\Theta}^*_{a^j(j)})$, and in Equation 8, $\hat{c}_j(\tilde{\Theta}^*_{a^j(j)})$. The requirement is that supplier $j$ must report her real cost function, so that $\hat{c}_j(\tilde{\Theta}^*_{a^j(j)}) = c_j(\tilde{\Theta}^*_{a^j(j)})$, in order to achieve maximal utility through the optimization of $\Theta^*$.

Next, we show the incentive compatibility of MGVC from another perspective. To do this, we first must define what a truthfully-implementable mechanism is in our setting:

Definition 1 (Truthfully-Implementable Mechanism). A mechanism is truthfully-implementable, if and only if, for any supplier $j \in J$, and any bidding vector reported by all suppliers $(C = \{\hat{c}(\cdot), i = 1, 2, \ldots, n. \hat{c}(\cdot) \neq c_j(\cdot)\})$, and for supplier $j$’s true cost profile $C' = [\hat{c}_j'(.), \forall i \neq j, \hat{c}_j'(\cdot) = c_j(\cdot)]$ (where $c_j(\cdot)$ is the real cost function of supplier $j$),

$$P_j(C') - c_j(\tilde{\Theta}(g_j(C'))) \geq P_j(C) - c_j(\tilde{\Theta}(g_j(C))).$$

In this expression: $P_j(C')$ is the payment to supplier $j$ when its true cost function is reported as $C'$. The
term \( P_j(C) \) is the payment to supplier \( j \) when the cost function is reported as \( C \) and may not be truthful. Also \( g_j(C) \) is the optimal allocation to supplier \( j \) when the costs are \( C \), and \( g_j(C') \) is similarly defined. Finally, \( \bar{\theta}(g_j(C)) \) is the optimal quality requirement vector under the allocation \( g_j(C) \), with \( \bar{\theta}(g_j(C')) \) defined in the same manner. A truthfully-implementable mechanism accurately maps from the supplier’s cost function and the quality requirement for the goods supplied, to the payment function in the presence of the supplier’s true costs and an allocation selected by the auctioneer.

**Proposition 2 (Truthfully-Implementable Mechanism Proposition).** MA-VCG is a truthfully-implementable mechanism.

A proof of this result is provided in the Appendix C. (See Appendix C.) It is routine to verify that truth-telling is an equilibrium strategy in any efficient mechanism (Krishna, and Perry, 2000; Mas-Colell, Whinston, and Green, 1995). In our case, we know that the MA-VCG mechanism implements the social choice function in Equations 1 and 4 as a dominant equilibrium strategy. Comparing this equilibrium strategy with how implementation is viewed in the context of the Bayesian-Nash equilibrium concept, the dominant equilibrium implementation is strong and robust. A rational agent who has a weakly dominant strategy will want to use it. But, unlike the equilibrium strategies in associated with a Bayesian-Nash equilibrium, a supplier does not need to correctly forecast the opposing supplier’s bids or types to justify his dominant strategy approach to bidding. The MA-VCG mechanism will be robust even if the suppliers have incorrect, and perhaps even contradictory, beliefs about the distribution of other suppliers’ cost functions.

One advantage of the dominant equilibrium strategy is that if the mechanism designer is an outsider (for example, the auctioneer or a government body in our model), then the designer will not need to know the probability density over realizations of the suppliers’ costs to successfully implement the social choice function via the allocation and payment functions.

**INDIVIDUAL RATIONALITY AND A BALANCED BUDGET FOR THE AUCTIONEER**

Up to this point in this article, we have assumed that each supplier has no choice but to participate in the auction. We limited the supplier’s judgment to choosing optimal actions within those allowed by the mechanism. However, in many cases, suppliers can voluntarily participate in combinational procurement
auctions. As a result, the social choice function that is to be implemented by a mechanism must not only be incentive-compatible. In addition, it must also satisfy the appropriate participation constraints or individual rationality to be successful. We note three stages in which participation constraints may be relevant under different assumptions.

The first of these stages occurs when we assume that suppliers may be able to withdraw from the auction and is called the *ex post* stage (Mas-Colell, Whinston and Green, 1995). This stage arises when all suppliers have announced their cost functions and an allocation (including the combinatorial assignment and quality requirements) has been chosen publicly. A related assumption is that suppliers cannot receive any positive utility by withdrawing from the auction. The second stage is the *interim* stage, which is defined as when all suppliers know their own cost functions for supply but have no precise information about the cost functions of other suppliers (Holmstrom and Myerson, 1983). The third stage is the *ex ante* stage, in which all suppliers do not know their own and other bidders’ cost functions exactly, but just the distributions information about them (Mas-Colell, Whinston and Green, 1995). We will next show that the MA-VCG mechanism is able to satisfy individual rationality in the *ex post* stage.

**Proposition 3 (The Ex Post Individual Rationality Proposition).** In MA-VCG, a supplier’s *ex post* individual rationality is assured.

**Proof:** Incentive compatibility in MA-VCG requires that the observed costs of a supplier in delivering goods of a required quality are the same as those truthfully reported by the supplier:

\[ c_j(\hat{\theta}^* a^{-i(j)}) = \hat{c}_j(\hat{\theta}^* a^{-i(j)}) , \] based on \( P_j = V(J) - V(J \setminus j) + \sum_{k \in a^{-i(j)}} \hat{c}_k(\theta_k) \). The *ex post* utility of supplier \( j \) can be expressed as \( u_j = V(J) - V(J \setminus j) \), which is non-negative. This completes the proof. □

We now will discuss the individual rationality of the auctioneer relative to her revenue. This can be expressed as follows in terms of the utility of a successful bidding combination, \( u^B \), when some suppliers win the opportunity to provide supplies:

\[ u^B = \sum_{j \in \text{winners}} v_j(\hat{\theta}^* a^{-i(j)}) - \sum_{j \in \text{winners}} P_j \]  \hspace{1cm} (9)

In this expression, the first term describes the value of all procured goods to the auctioneer associated with the corresponding optimal quality requirements, reduced by the relevant payments to suppliers in the
second term. Note that using Equations 4, 5 and 6, we can rewrite Equation 9 as:

\[ u^B = \sum_{j \in \text{winners}} v(\hat{\theta}_a^{+}(j)) - \sum_{j \in \text{winners}} [V(J)-V(J \setminus j) + \sum_{k \in \text{JV}} c_{jk}(\theta_k)] \]

\[ = V(J) - \sum_{j \in \text{winners}} [V(J)-V(J \setminus j)] \]

\[ \sum_{j \in \text{winners}} [V(J)-V(J \setminus j)] \] is the total surplus of all of the winning suppliers, while \( V(J) \) is the social surplus. However, we cannot be certain that the value of the utility, \( u^B \), is greater than or equal to zero. In some extreme cases, \( V(J) \) can be negative, but the social surplus optimization that we specified in Equations 1 and 4 will still work. For example, a government auctioneer may wish to procure something or launch some projects, such as a swimming stadium for the Olympic Games in Beijing or a land reclamation project for low-income housing in Minneapolis. When the visible utility of these projects is less than the cost, the government’s procurement approach typically is to minimize the extent of the negative benefit-cost difference. In addition, it may be appropriate to think of Equation 10 as a non-negativity condition for the auctioneer’s individual rationality, if break-even or better is required. When the aggregate social surplus is greater than the total winners’ surplus, the auctioneer will benefit from the difference.

Whenever gains from auction-based trade are possible, but not certain, there is no ex post efficient social choice function that is Bayesian incentive-compatible and will satisfy the participation constraints (Myerson and Satterthwaite, 1983). Under the conditions of the theorem, the presence of both private information and voluntary participation makes it impossible to achieve ex post efficiency. In other words, no ex post efficient social choice function will be implementable via the dominant strategies with participation constraints.

In government procurement auctions, the auctioneer typically emphasizes fairness, transparency, bidding incentives and social welfare to a greater extent than we typically observe in the private sector. Visible profit may not be as critical a factor in the social planning context. This means that it may be inappropriate to over-emphasize the assumption of individual rationality, when there is a quasi-participation constraint that permits participation with negative utility.

Another desired property of any mechanism for combinatorial procurement auctions is a balanced
budget. It requires that within the system, no resources are wasted, and external to it, no subsidies are contributed. In effect, to achieve a balanced budget, it is necessary for the sum of the transfers among all participating agents in a combinatorial auction to be zero. The impossibility theorem of Green and Laffont (1979) suggests that there is no mechanism that is truthfully-implementable in dominant strategies and is also ex post efficient. This requires that the social choice function will satisfy balanced budget constraint. However, one special case occurs in which there is one agent (which, in our context, could be the procuring auctioneer) whose preferences are known, and thus, holds no private information. We call the auctioneer Agent 0, and assume there are $n$ other agents whose preferences are private information, for a total of $n + 1$ agents. In the presence of Agent 0, ex post efficiency is compatible with any payment function, $P_j$, $j = 1, 2, ..., n$ for the $n$ agents with private information, if we set the payment to Agent 0 as $t_0 = -\sum_{j \neq 0} P_j$.

In our MA-VCG mechanism, the payment function for the auctioneer exactly meets the condition required above. Moreover, in multi-attribute combinatorial procurement auctions with $n + 1$ agents (1 auctioneer and $n$ suppliers), all suppliers who hold private information will receive non-negative payments from the auctioneer, and a balanced budget will be obtained.

THE IMPORTANCE OF FUNCTIONAL FORM FOR THE PAYMENT FUNCTION

Up to this point, we have seen that the social choice function that we specified is truthfully-implementable via dominant strategies for bidding participation in a combinatorial procurement auction. But are these the only social choice functions that will satisfy Equation 1 and be truthfully-implementable? Our next proposition considers this issue in more detail.

**Proposition 4 (The Payment Functional Form Proposition).** The social choice function (via the allocation function in Equation 1 and the payment function in Equation 4) is truthfully-implementable in dominant strategies only if the payment is specified according to a workable functional form.

A proof of this result is provided in the Appendix D.

Our MA-VCG mechanism can also effectively treat a number of specific cases. Consider an auctioneer who wishes to procure heterogeneous goods, and permits suppliers to bid for any combination (subset) of these goods. Supplier $j$ bids according to $c_j(.) = \{c_{jk}(.)\}, \forall k \in B$, with functional form $c_{jk}(.) = c_{jk} \theta_k$. Here, $\theta_k$
\( \in [\underline{\theta}, \overline{\theta}] \) is a quality requirement determined by the auctioneer after completion of bidding. As before, \( c_{jk} \in [c_{jk}^L, c_{jk}^R] \) denotes the supplier’s cost function type. Supplier \( j \)'s bidding vector can be expressed as \( c_{j}(\cdot) = \{c_{jk}\} \), for all \( k \in B \). For convenience, we will use the following expression: \( f'(c_{j}, c_{-j}) = (a'(c_{j}, c_{-j}), \Theta'(c_{j}, c_{-j})) \).

The left-hand side, \( f'(c_{j}, c_{-j}) \), denotes the optimal assignment of combinations and the quality requirement for supplied goods, when supplier \( j \) bids \( c_{j} \) in the presence of the other \( n-1 \) suppliers bids, \( c_{-j} \). In these cases, the social choice function (including the payment function and the allocation function) is truthfully-implementable in dominant strategies only if payment satisfies a functional form like Equation 4. (For additional analysis to support this observation, see Appendix E.)

**ENHANCING THE AUCTIONEER’S PROFIT WITH A REVISED MECHANISM**

If some truthfully-implementable mechanism can enhance the profit that the auctioneer can earn in settings where profitability is preferred to social welfare, then this agent likely will choose to use the mechanism that enables him to achieve the greatest utility or financial advantage. In this section, we explore the construction of an alternate social choice function that involves a revised payment function that is advantageous to the auctioneer, in the presence of the same social goals as in the allocation function in Equation 1.

We define another improved payment function:

\[
P_{j}(\hat{c}_{j}|c_{j}) = V(J) + \hat{c}_{j}(\hat{\theta}_{a^{-1}(j)}) - V(J_{-j}, \overline{c}_{j})
\]  

(11)

where

\[
V(J_{-j}, \overline{c}_{j}) = \sum_{i \neq j} [v_{i}(\overline{\theta}_{a^{-1}(i)}) - \hat{c}_{i}(\overline{\theta}_{a^{-1}(i)})] + [v_{j}(\overline{\theta}_{a^{-1}(j)}) - \overline{c}_{j}(\overline{\theta}_{a^{-1}(j)})]
\]  

(12)

is the social surplus. Supplier \( j \)'s cost function is \( \overline{c}_{j}(\cdot) \), and she submits a bid of \( \overline{c}_{j}(\cdot) \) to the auctioneer, while the other supply bid \( \hat{c}_{i}(\cdot) \). In this instance, \( a^{-1}(j) \) denotes the allocation that the auctioneer makes (in terms of the combination of good supplied and the quality standard that they meet) to supplier \( j \).

If she wins, the *ex post* utility of supplier \( j \) is given by:
\[ u_j(c_j|\bar{c}_j) = V(J) + \hat{c}_j(\bar{\theta}_{a^{-1}(j)}) - V(J_{-j}, \bar{c}_j) - c_j(\bar{\theta}_{a^{-1}(j)}) \]
\[ = \sum_{i \neq j} [V_i(\bar{\theta}_{a^{-1}(i)}) - \hat{c}_i(\bar{\theta}_{a^{-1}(i)})] + [V_j(\bar{\theta}_{a^{-1}(j)}) - c_j(\bar{\theta}_{a^{-1}(j)})] - V(J_{-j}, \bar{c}_j) \]  

(13)

By implementing a similar analysis of the original mechanism’s incentive compatibility, we can show that supplier \( j \) must bid her real cost function, \( c_j(.) \), in the revised mechanism to maximize the value of her ex post utility in Equation 13. It is evident from the form of the equation that her revelation of her cost function is unaffected by \( V(J_{-j}, \bar{c}_j) \). In the improved mechanism, suppliers will have an incentive to submit bids that reflect their true underlying cost functions.

Next, we define the optimal value of the supplier’s bid, \( \bar{c}_j(.) \), in terms of the auctioneer’s revenue:

\[ \bar{c}_j(.) = \arg \min_{c_j(.) \in C_j(.)} V(J_{-j}, c_j) \]  

(14)

\( C_j(.) \) reflects the range of possible bids (and underlying costs) of the supplier. The reader can think of \( \bar{c}_j(.) \) as the bid level for providing a package of supplies that is associated with indifference on the part of supplier \( j \). If supplier \( j \) has true cost \( \bar{c}_j(.) \), then she will obtain utility 0 from the auctioneer’s effort to minimize procurement cost. We assume that the auctioneer knows the range of cost functions for all of the potential suppliers, but only will come to know their real cost functions after the bidding is done. Because the revised mechanism is incentive-compatible, our assumption about the auctioneer’s knowledge of the true cost functions of all of the suppliers is reasonable. Define the equilibrium state as the state in which suppliers bid their true cost functions. Equilibrium can be achieved in the incentive-compatible revised mechanism. So, in the equilibrium state of the revised mechanism, we can rewrite Equation 13 as:

\[ u_j(c_j|\bar{c}_j) = V(J) - V(J_{-j}, \bar{c}_j) \]  

(15)

This expression is non-negative because the suppliers’ ex post participation constraint is satisfied. \( V(J_{-j}, \bar{c}_j) \) is the social surplus for a specific supplier \( j \) who has cost function \( \bar{c}_j(.) \). Considering the definition of \( \bar{c}_j(.) \), \( V(J_{-j}, \bar{c}_j) \) is the least maximal social surplus with supplier \( j \)’s participation, given the other suppliers’ bids. For this reason, Equation 15 is non-negative.
We argued earlier that the ex post utility of supplier $j$, $u_j(c_j | \bar{c}_j) = V(J) - V(J_{-j}, \bar{c}_j)$, is positive. To show this more formally, we first need to examine two cases in which supplier $j$ is selected as one of the winners by the auctioneer in a procurement combinatorial auction.

- **Case 1 (Supplier’s True Cost Is Better than Expected).** If the supplier $j$’s true cost function, $c_j(\cdot)$, is better than what the auctioneer thinks, as represented by the indifference level $\bar{c}_j(\cdot)$, then
  \[ V(J_{-j}, c_j) = V(J) > V(J_{-j}, \bar{c}_j). \]
  That is a restatement of our prior assertion: that the ex post utility of supplier $j$ is positive in Equation 15.

- **Case 2 (Supplier’s True Cost Matches the Auctioneer’s Expectation).** If Supplier $j$’s true cost function $c_j(\cdot)$ is the same as $\bar{c}_j(\cdot)$, then the utility associated with combinatorial-based procurement will be zero for the supplier. Based on the definition of $\bar{c}_j(\cdot)$, it is impossible that $c_j(\cdot)$ is worse than $\bar{c}_j(\cdot)$.

If supplier $j$ is not selected as one of the winners, then the utility will also be zero. Consequently, the utility that obtains from the supplier’s participation in the auction, as indicated in Equation 15, will be non-negative, which means that ex post individual rationality will hold.

We also know that $V(J_{-j}, \bar{c}_j) \geq V(J \setminus j)$ \( \forall j \) will be true. That is because the level of social welfare that obtains with the participation of suppliers whose bids are made at the indifference level (yielding a zero utility)—including bidder $j$’s bid—will be greater than or equal to the social welfare that accrues without her participation. By comparing the payment function for the original mechanism in Equation 4 with the revised mechanism in Equation 11, we can see that the payment that will be made to any supplier $j$ will be less for the latter. This means that the auctioneer can obtain the same quality products or services for less money. We should not view the revised mechanism as being better than in the original mechanism in social welfare terms, however. What is happening is that the same amount of surplus from suppliers ends up in the hands of the auctioneer, with the result that there is no increase in total social welfare. Only the auctioneer’s utility is changed by the revised payment scheme.
By choosing another optimal $\bar{c}_j(.)$ based on the conditional expectation, we hope to show that the revised mechanism is the one that maximizes expected utility for all incentive-compatible mechanisms in our combinatorial procurement auction setting. The purpose of applying the conditional expectation is to depict settings in which a supplier who knows her own cost function may not know the cost functions of the other suppliers, as in the *interim stage* in mechanism design theory discussed earlier. Therefore, the only effective way for the supplier to forecast her revenue is to evaluate other suppliers’ cost functions based on what she knows about her own.

To illustrate these ideas, we first define the *expected utility* of supplier $j$ with cost function $c_j(.)$ in any incentive-compatible, participation-constrained and efficient mechanism in our environment as:

$$U_j(c_j) = E_{c_j}[u(c_j)] = t_j(c_j) - p_j(c_j(\tilde{\theta}_{a^{-j}(j)}) \cdot c_j(\tilde{\theta}_{a^{-j}(j)})$$  \hspace{1cm} (16)

Here, $t_j(c_j)$ is the expected payment value with supplier $j$’s cost function $c_j(.)$. But supplier $j$ is not aware of other suppliers’ cost functions. Thus, the payment value will not be deterministic in the interim stage for supplier $j$. Further, her revenue from the combinatorial auction must be evaluated in expected value terms, as $p_j(c_j(\tilde{\theta}_{a^{-j}(j)}))$. This is the probability in which supplier $j$ is assigned an allocation based on her bidding combination that meets the standard that the auctioneer sets for the multi-attribute quality vector of the supply goods, $\tilde{\theta}_{a^{-j}(j)}$. Since the amount the supplier is willing to pay via her cost function also reflects her expected utility, we can employ supplier $j$’s cost function $c_j(.)$ in the revised mechanism, to obtain the optimal value of supplier utility, $U_j(.)$, as follows:

$$U_j(c_j|\bar{c}_j) = E_{c_j}[u(c_j|\bar{c}_j)] = t_j(c_j|\bar{c}_j) - p_j(c_j(\tilde{\theta}_{a^{-j}(j)}) \cdot c_j(\tilde{\theta}_{a^{-j}(j)})$$  \hspace{1cm} (17)

$t_j(c_j|\bar{c}_j)$ is the expected payment to supplier $j$ established on the basis of true cost function, $c_j(.)$, in the equilibrium state of the application of the revised combinatorial auction mechanism.

Second, choose the conditionally-expected optimal value of $\bar{c}_j(.)$ is based on:

$$\bar{c}_j(.) = \arg \min_{c_j(.) \in C_{j(\bar{c}_j)}} E_{c_j}[V(J_{-j}, c_j)]$$  \hspace{1cm} (18)
Applying Krishna’s (2000) expected utility equivalence theorem, for any incentive-compatible, participation-constrained and efficient mechanism which has the same social goal, we obtain the following equality, which is known as the revenue equivalence theorem:

\[ U_j(\overline{c}_j) - U_j(c_j) = U_j(\overline{c}) - U_j(c_j) \]  

(The Revenue Equivalence Theorem)  

(19)

This theorem states that all incentive-compatible, participation-constrained and efficient mechanisms with the same social goal will generate the same difference in expected utility corresponding to different cost functions for the suppliers. The left hand side of Equation 19 denotes the difference in expected utility of supplier \( j \) based on different cost functions, i.e., \( c_j(.) \) and \( \overline{c}_j(.) \), in the revised mechanism. The right hand side is the difference in expected utility of supplier \( j \) based on \( c_j(.) \) and \( \overline{c}_j(.) \), which applies to any other specific incentive mechanism with the same social goal as included in our model (based on Equations 1 and 4).

Rearranging terms in Equation 19, we obtain:

\[ U_j(c_j) - U_j(c_j) = U_j(\overline{c}_j) - U_j(\overline{c}) \]  

(20)

Individual rationality requires that \( U_j(\overline{c}_j) \geq U_j(\overline{c}) \) \( \forall j \). Consequently, it must also be the case that \( U_j(c_j) \geq U_j(c) \) \( \forall c_j(.) \), which, in turn, means that \( t_j(c) \leq t_j(c) \) \( \forall c_j(.) \). Thus, we conclude that the revised mechanism, with its optimal conditionally-expected value of \( \overline{c}_j(.) \), will maximize the expected utility of the auctioneer by minimizing all of the suppliers’ expected payments with the same allocation rules and the same allocation outcome—in other words, the same combinatorial assignment of goods at the criterion level of quality demanded by the auctioneer. (Appendix F provides a numerical illustration of the base and the revised mechanisms to aid the reader’s understanding of the concepts.)

DISCUSSION

We next consider several additional issues associated with the performance of electronic multi-attribute combinatorial auctions in practice, and relate them back with our results for the MA-VCG mechanism design that we have proposed.
Mechanism Robustness and Contract Completeness. The most important issues in combinatorial auction design are the concerns of efficient allocation, revenue maximization, truthfulness in bidding and the prevention of collusion. Constraints on participation and the balancing of the budget should be regarded as issues of second-order importance. The application of combinatorial auctions for FCC spectrum, mobile phone licenses, TV franchises, and government services shows that a good design should be robust in terms of its market performance quality to the vagaries of the changes in the content of the underlying exchange transactions and the participants who wish to make them. In addition to the economic outcomes that we have been discussing throughout this article, the criterion of mechanism robustness suggests that the auction market mechanism should be easy for market participants to master, capable enough in its functionality to engender the participation of different kinds of trading agents, and flexible enough to be effective in the presence of significant changes in the marketplace (e.g., related to demand and supply, the need for trading agent learning, pressure for changing payment rules, and so on).

In combinatorial procurement auctions, the auctioneers hope to use the market mechanism to effectively procure goods or services at an attractive price, with reasonable operating costs and acceptable risks. The special characteristic of procurement combinatorial auctions is the evaluation of multiple attributes of the trade items that the auctioneer wants its suppliers to provide. Underlying the suppliers’ bids are real contracts. These commit the supplier to full implementation, including the delivery of the trade items at the specified service level or quality prescribed in the contract. However, when the procurement contracts pertain to goods or services that are not true commodities or are not commodity-like in their features, then there always is a possibility that the quality of the auction market mechanism may not be the key factor. Instead, low bids and best prices may fail to accurately reflect the risks to the auctioneer that are inherent in the non-contractible elements of exchange (Hart and Moore, 1990). So it is appropriate for the reader to recognize the critical importance of the implementability of a given auction market mechanism relative to the desired economic and social welfare outcomes (Nault and Tyagi, 2002).

Consider the contrast with the issues associated with forward auctions, too. The suppliers and the auctioneer will know about the goods or services to be procured, and since there is no need to consider current excess or short supplies with the suppliers, the bidding will focus on competition among suppliers
and their prices. By combining the benefits of the VCG mechanism with multi-attribute evaluation, our MA-VCG mechanism provides a useful theoretical extension to prior research, as well as the treatment of practical issues that arise in combinatorial auctions for procurement.

**Reserve Prices.** Special consideration needs to be given in combinatorial procurement auction situations where there is the possibility of collusion. If bidders, acting as suppliers in a procurement auction, collude to cause a very high supply price or too low a quality standard, then they will gain profit that will cause financial harm to the auctioneer. So the auctioneer must take special precautions to ensure that there is an incentive compatible auction mechanism to prevent such undesirable behavior. Prior theoretical models of collusion among bidders involve both non-cooperative and cooperative game theory (e.g., Kagel, 1995; Klemperer, 2002). In the theory of auctions also in practice, however, the usual method for combating collusion is for the auctioneer to set a reserve price. However, even in forward auctions, many of problems that have been reported have been aggravated by the failures of the auctioneer to set the proper reserve prices (e.g., Klemperer, 2002 and 2004; Krishna, 2002). When the reserve price is inadequate, it increases the incentives for predation and may encourage collusion that will not be in all bidders’ interests. On the other hand, excessively high reserve prices are often unattractive. State enterprises or government procurement agents that want to sell goods may oppose implementing a reserve price because, if the reserve price is not met, then the trade item will not be sold and the auction will be seen as having failed.

In combinatorial procurement auctions, setting reserve prices becomes more complex. There are two issues that deserve comment. The first issue is that, because of computational complexity, reserve prices associated with different combinations of trade items in a combinatorial auction will be hard to specify. This will be especially true in reverse auction settings, where the buyer is the procuring auctioneer. The second issue is involves the nature of bidding in multi-attribute combinatorial auctions. It is hard to set reserve prices associated with the highest prices and the lowest quality standards for different combinatorial bundles properly. The difficulties come due to the lack of information from the start about the suppliers’ cost functions for supplying the combinatorial bundles at different price and quality standard combinations. Another associated difficulty is that there is no means prior to the auction to cost effectively specify the auctioneer’s own valuation for the quality standards and prices that are to be presented in the auction.
To address the reserve price issues that we have just raised, our MA-VCG mechanism models the bidders’ cost functions and auctioneer’s valuation function first. Because of the difficulties associated with setting appropriate reserve prices, “truthful bidding” on the part of the suppliers is of crucial importance to the auctioneer. This will permit the procurement auction to proceed successfully. We proposed that “truthful bidding” should be modeled in terms of the information revelation by suppliers of their true cost of delivering the agreed upon goods or services, should any one of them win the auction. This kind of report about the underlying costs is supported in terms of its theoretical contents by the literature that deals with behavioral incentives on the part of agents and their revelation of asymmetric information. We recognize that the cost function associated with follow through on a winning procurement contract is easier to estimate and report than “quality-and-price” bids for any supplier, even though it still is somewhat different from bidding, as a means of direct revelation. In some cases, when suppliers are reluctant to report their true costs (e.g., when the procurement setting has the flavor of a repeated game over time, with some suppliers pursuing strategies that may force their competitors out of the market in the long run), then the equivalent “quality-and-price” bidding auctions should be constructed based on VCG mechanism design, where the uniqueness of its incentive-compatibility property is widely accepted.

A Final Word on the Applicability of MA-VCG. In what kinds of settings will the proposed MA-VCG mechanism be most well suited? The key thing to keep in mind is that the mechanism is designed to maximize the total utility of the system, which we have described in social welfare terms. This perspective takes into account the point of view of the auctioneer, who may represent government procurement, and the trade goods and services bundle suppliers, who may be either public or private entities. Our model formulates this in terms of the total utility of the system as the sum of the differences of the valuations of all the procured items based on the specific quality requirements and corresponding costs. This formulation considers the aggregate value adjusted by the total costs. Compared with the optimization goal of the traditional VCG mechanism—which adds the valuations of all of the agents together—our MA-VCG mechanism tries to balance the utility tradeoff inherent in the government auctioneer’s costs relative to the payment revenues of the winning suppliers. Adding the separate valuations does not yield an allocatively efficient solution in our case. The incentive-compatibility and the ex post participation
constraint properties of MA-VCG imply that it will preserve the advantageous properties of VCG in multi-attribute combinatorial procurement auctions, while considering economic tradeoffs. In other words, the goal of optimizing all winning bidders’ utility in the VCG mechanism becomes broader under MA-VCG. The goal shifts to including the optimization of joint utility, so that both the auctioneer and the winning suppliers are considered. In our view, the main difference that drives the necessity of this shift in optimization perspective is that multi-attribute combinatorial bidding and evaluation are bound up with both the suppliers’ cost functions and the buyer/auctioneer’s value function. Moreover, the original optimization goal of the VCG mechanism fails to effectively take into account aggregate social welfare.

CONCLUSIONS

We proposed a new mechanism, MA-VCG, for multi-attribute combinatorial procurement auctions, by expanding the mechanisms attributable to Vickrey (1961 and 1962), Groves (1973) and Clarke (1973).

Contributions

Our MA-VCG mechanism, as we have shown in this article, is incentive-compatible, quasi-participation constrained, budget-balanced and efficient in a setting with quasi-linear preferences. We modeled a setting in which the auctioneer has no private information. This setup meets the criteria required for ex post efficiency for a dominant strategy incentive-compatible mechanism. Although the ex post participation constraint for the auctioneer is not satisfied in our MA-VCG mechanism, however, all suppliers still are individually rational in their bids, with valuations up to the limit of their costs.

We also explored the theoretical reasons and practical background for why our MA-VCG mechanism works the way that it does. We noted that when the budget is balanced in our auction setting, the transfers in value via auction procurement payments for all suppliers are supplied by the auctioneer. We also showed that the payment function must have a unique form, like the payment function in Clarke mechanism, so that it is able to satisfy both the dual countervailing criteria of efficiency and incentive-compatibility.

In consideration of the desirability of capturing greater value from the auctioneer’s viewpoint, we also proposed a revised payment function based on the original mechanism to achieve higher revenue for the auctioneer. Intuitively, the change that we made to the payment function cannot increase the aggregate social surplus available to the auction agents. However, it does shift some amount of value from the
winning suppliers to auctioneer, who is procuring supplies. For each winning supplier, we found that the amount of decreased profit, compared with original mechanism, is $V(J^-,c_i) - V(J^-)$.

For the *interim stage* of our MA-VCG mechanism, we applied Krishna’s (2002) version of the well known revenue equivalence theorem, and showed the maximal expected revenue property of the revised mechanism. Basically, the payment scheme—with its optimal conditionally-expected value of $\bar{c}_j(.)$—will maximize the expected utility of the auctioneer by minimizing all suppliers’ expected payments with the same combinatorial assignment of goods at the same criterion level of quality.

**Limitations and Future Research**

There are three primary limitations of the current work. First, the winner determination problem for combinatorial auctions suggests that for problems involving many suppliers and different possible bidding combinations, the feasibility and complexity of these allocations will be critical limiting factors for the viability of any proposed mechanism. We assumed that there are enough suppliers to participate in the procurement auction, so that an allocation outcome, in terms of the subsets of all goods that are procured, will always exist. This permitted our MA-VCG mechanism to provide a theoretical solution (if not a real-time one) for the optimization problem to ensure the best allocation outcome for the suppliers and the auctioneer, in view of the applicable payment rules. And thus we put the computational problems aside—for the moment, at least. But the reader should bear in mind that the tensions between game-theoretic solutions and computational solutions become critical when consider applying the MA-VCG mechanism design to practical managerial problems, such as real world supply chain procurement and forward combinatorial auctions.

Second, another controversial assumption in our model is that the suppliers are only able to supply one specific combination of goods via the MA-VCG mechanism. The reader can think of each supplier as an “all-or-nothing supplier,” who hopes to win with her bidding supply combination only, or have no auction transaction. It turns out that this assumption eases the complexity of valuation for the auctioneer’s effort to solve its social choice problem and make an efficient and incentive-compatible procurement supply allocation. Complexity here will determine how much computation will be needed to represent the complete preferences of the suppliers, so that the auctioneer is able to determine the appropriate allocations.
We should also point out that it is possible that incentive compatibility will break down if the allocations to suppliers are not complete. We are currently working through the details of this issue.

This brings up another related issue involving the extent of communication (e.g., revelation of the suppliers’ cost functions) that is required between the suppliers and the auctioneer to yield an optimal outcome with the mechanism. The idea of an all-or-nothing supplier is more reasonable in supply chain procurement settings, where the auctioneer or buyer is subject to considerations about the transaction costs and risks inherent in spreading orders across multiple suppliers (Clemons, Reddi and Row, 1993), and the possibility of supply and demand shocks that may require subsequent procurement renegotiation (Kauffman and Mohtadi, 2004). The assumption of an all-or-none supplier is lest realistic in forward combinatorial auctions: suppliers are always capable of supplying certain goods given sufficient lead time.

Third, we formulated our model as a “one-shot” auction. This requires a fairly strong assumption about the preferences of the suppliers: that they know them well enough from the start to be able to bid effectively, ignoring the need for a process to elicit their preferences and values. In contrast, iterative combinatorial auctions allow suppliers to submit different bids at different times. This makes sense because some bidding combinations may have “package-contingent” valuation properties. Consider the example we discussed earlier of airline landing and takeoff rights at an airport. The airlines’ valuation for different combinations of landing and takeoff slots is likely to depend on the range of different feasible outcomes. Too many morning slots in a package may diminish the value of any individual morning slot. Indeed, in a package with slots that cover the entire travel day, a few morning slots ought to be of higher value.

Thus, we see that, relative to our MA-VCG mechanism, a one-shot combinatorial auction assumes that suppliers will know and be able share information about their complete valuations for different combinations of the procurement goods, based on all of the different possible allocations. Iterative combinatorial auctions may do better by allowing suppliers to adapt their valuations and adjust their bidding actions during the course of the auction based on relevant information that is fed back by the auction mechanism (e.g., other suppliers’ bids, the current provisional allocations, etc.). Another issue is the extent of collaboration and cooperation among suppliers in electronic combinatorial procurement auctions, which may make some agents better able to acquire private information from their auction
competitors.

Two other issues that we have not considered which deserve additional attention relative to supply chain procurement auction markets include market transparency, supplier identification and information privacy protection. Although these are practical concerns, we know from the historical record as we have observed it the financial markets around the world (e.g., the “Big Bang” City of London stock market deregulation in the late 1980s, and transparency differences among the market mechanisms in New York, Tokyo and Toronto financial markets) that these issues will be critical in the one-shot combinatorial auctions context. Nevertheless, we expect that these issues in the procurement e-auctions can be probably be handled to some extent by well-designed iterative combinatorial auctions. We expect to address some of these issues in future research.

REFERENCES


Appendix A. Definitions of Key Technical Terms Used in the MA-VCG Mechanism Analysis

<table>
<thead>
<tr>
<th>Key Construct</th>
<th>Definition</th>
</tr>
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<tbody>
<tr>
<td>Allocative efficiency</td>
<td>In the supplier bidding context, to direct orders from the auctioneer to those suppliers who value delivering the supplies at the quoted price the highest.</td>
</tr>
<tr>
<td>Allocation function</td>
<td>The optimization problem that the MA-VCG mechanism solves, by determining which suppliers’ bids are accepted for procurement.</td>
</tr>
<tr>
<td>Allocation outcome</td>
<td>An optimal solution to the winners’ determination problem</td>
</tr>
<tr>
<td>Balanced budget</td>
<td>Occurs when no resources are wasted no subsidies are needed in a combinatorial auction, with the sum of the transfers among all participating agents equal to zero.</td>
</tr>
<tr>
<td>Bayesian-Nash equilibrium</td>
<td>A Nash equilibrium at the interim stage, where each supplier selects a best response against the average best responses of the competing suppliers, based asymmetric information about the cost functions of the different suppliers (Econport, 2005).</td>
</tr>
<tr>
<td>Bidding combination</td>
<td>A combination of procurement goods that a supplier offers to the bidding auctioneer.</td>
</tr>
<tr>
<td>Direct revelation mechanism</td>
<td>Auction mechanism, in MA-VCG setting, such that bids of any supplier will contain only two kinds of information: bidding combination and corresponding cost function.</td>
</tr>
<tr>
<td>Dominant strategy solution</td>
<td>Suppose a supplier evaluates the available bidding strategies for procurement and chooses the one that gives her the highest utility. When she uses this strategy to respond to all opposing suppliers’ bidding strategies, it is a dominant strategy.</td>
</tr>
<tr>
<td>Equilibrium state</td>
<td>In MA-VCG, state in which all suppliers report their true cost functions to auctioneer.</td>
</tr>
<tr>
<td>Ex post stage</td>
<td>All suppliers announce their cost functions and know other suppliers’ cost functions.</td>
</tr>
<tr>
<td>Feasible allocation</td>
<td>Solutions to winner determination problem in procurement combinatorial auctions.</td>
</tr>
<tr>
<td>Incentive-compatibility</td>
<td>A modeling constraint that encourages the supplier to report a true cost function to the auctioneer and other suppliers; leads to the maximization of the supplier’s utility.</td>
</tr>
<tr>
<td>Individual rationality</td>
<td>Modeling constraint to ensure supplier participation in combinatorial procurement auction is voluntary, based on the level of utility that is obtained by the supplier.</td>
</tr>
<tr>
<td>Interim stage</td>
<td>Occurs when all suppliers know their own cost functions for supply but have no exact information about the cost functions of other suppliers.</td>
</tr>
<tr>
<td>Maximal expected revenue</td>
<td>Under the assumptions of the revenue equivalence theorem, the expected revenue of a combinatorial auction will be equal to the expected marginal revenue of the supplier with the winning bid (Bulow and Roberts, 1989, Klemperer, 2003 and 2004).</td>
</tr>
<tr>
<td>property</td>
<td></td>
</tr>
<tr>
<td>Multiple attributes</td>
<td>More than one descriptor for an auction good (e.g., rare coin year and collector grade).</td>
</tr>
<tr>
<td>Pareto efficiency</td>
<td>Pareto efficiency occurs when there is no reallocation of supplies that can improve the utility of any supplier without decreasing the utility of another suppliers.</td>
</tr>
<tr>
<td>Payment function</td>
<td>Procurement combinatorial auction mechanism rules that determine how much the auctioneer pays to procure supplies from the supplier who wins the auction.</td>
</tr>
<tr>
<td>Social choice function</td>
<td>Composite of allocation function (Equation 1) and payment function (Equation 4).</td>
</tr>
<tr>
<td>Supplier type</td>
<td>A supplier’s cost function is the primary determinant of supplier type (e.g., high cost or low cost supplier), and determines the supplier’s bids.</td>
</tr>
<tr>
<td>Truthfully-implementable</td>
<td>Auction mechanism that creates incentive-compatibility for the suppliers in a combinatorial auction to reveal their true supply cost functions to the auctioneer.</td>
</tr>
<tr>
<td>mechanism</td>
<td></td>
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<tr>
<td>Vickery-Groves-Clarke</td>
<td>Combinatorial auction mechanism where suppliers simultaneously submit sealed bids to value rights to supply all sets of procurement goods. An auctioneer uses suppliers’ bids to construct efficient allocations. Payments determined to give each bidder a payoff equaling the incremental surplus that he brings to the auction (Ausubel, 2004).</td>
</tr>
<tr>
<td>mechanism</td>
<td></td>
</tr>
<tr>
<td>Weakly-dominant strategy</td>
<td>Supplier strategy for bidding that can be used in the absence of knowledge or beliefs about the cost functions of other suppliers.</td>
</tr>
<tr>
<td>Winner determination problem</td>
<td>The problem of deciding via the auction mechanism who will win the right to supply the procurement goods to the auctioneer.</td>
</tr>
</tbody>
</table>
Appendix B. Definitions for Key Modeling Notations

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>DEFINITION</th>
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<tbody>
<tr>
<td>( A )</td>
<td>The set of all feasible allocations made by the auctioneer</td>
</tr>
<tr>
<td>( A )</td>
<td>A specific optimal allocation for procurement supplies from the suppliers</td>
</tr>
<tr>
<td>( a^*(j) )</td>
<td>The allocation of supply goods for supplier ( j )</td>
</tr>
<tr>
<td>( B; B^k = { c_{jk}(\cdot) }, \forall k \in B )</td>
<td>The set of possible bidding combinations by all suppliers; and by one specific supplier ( j )</td>
</tr>
<tr>
<td>( \hat{c}<em>j(\cdot) ; \hat{c}</em>{jk}(\cdot) )</td>
<td>Supplier ( j )’s bidding cost function for her bidding combination; and for one element of the combination, goods ( k )</td>
</tr>
<tr>
<td>( c(\cdot) ; c_{jk}(\cdot) )</td>
<td>Supplier ( j )’s real cost function for her bidding combination; and for goods ( k )</td>
</tr>
<tr>
<td>( p_j(c(\cdot); \hat{c}_{jk}(\cdot)) )</td>
<td>Probability supplier ( j ) is assigned her bidding combination with quality requirements vector ( \hat{c}_{x^*(j)} )</td>
</tr>
<tr>
<td>( P_j(\hat{c}_j) \ (P_j(\hat{c}_j</td>
<td>F_\cdot)) )</td>
</tr>
<tr>
<td>( t(c) )</td>
<td>Expected payment value to supplier ( j ) with cost function ( c(\cdot) ) in equilibrium state in any incentive-compatible, participation-constrained and efficient mechanism</td>
</tr>
<tr>
<td>( t_j(c_j</td>
<td>F_\cdot) )</td>
</tr>
<tr>
<td>( \theta_k )</td>
<td>Quality requirements for a bundle of goods, ( k )</td>
</tr>
<tr>
<td>( \hat{\theta}<em>{x^*(j)} \ (\hat{\theta}</em>{x^*(j)}</td>
<td>F_\cdot) )</td>
</tr>
<tr>
<td>( \Theta ; (\Theta^*) )</td>
<td>Quality (Optimal quality) requirements that must be met for all goods procured by the auctioneer</td>
</tr>
<tr>
<td>( u_j(\Theta^*, P_j) )</td>
<td>Supplier ( j )’s utility function expressed in terms of the quality requirements of the procurement goods and the payment function</td>
</tr>
<tr>
<td>( u_j(c_j</td>
<td>F_\cdot) )</td>
</tr>
<tr>
<td>( U_j(c) )</td>
<td>Expected utility of supplier ( j ) with cost function ( c(\cdot) ) in any incentive-compatible, participation-constrained and efficient mechanism</td>
</tr>
<tr>
<td>( U_j(c_j</td>
<td>F_\cdot) )</td>
</tr>
<tr>
<td>( v(c_k) )</td>
<td>Utility function of the auctioneer for goods ( k ) with quality ( \theta_k )</td>
</tr>
<tr>
<td>( V(J) )</td>
<td>Maximal social surplus for all procured goods</td>
</tr>
<tr>
<td>( V(J \setminus j) )</td>
<td>Maximal social surplus based on all suppliers’ participation</td>
</tr>
<tr>
<td>( V(J \setminus j, \bar{F}_j) )</td>
<td>Maximal social surplus with supplier ( j ) not participating</td>
</tr>
</tbody>
</table>

Appendix C. Proof of Proposition 2 (The Truthfully-Implementable Mechanism Proposition)

From Equation 1 in the main text of the paper, we know that:

\[
\sum_{i=1}^{n} [v(\hat{\theta}(g_i(C))) - \hat{c}_i(\hat{\theta}(g_i(C))))] \geq \sum_{i=1}^{n} [v(\hat{\theta}(g_i(C))) - \hat{c}_i(\hat{\theta}(g_i(C))))] 
\]  
(C1)

We also note that for all \( i \neq j \):

\[
v(\hat{\theta}(g_i(C))) - \hat{c}_i(\hat{\theta}(g_i(C)))) = v(\hat{\theta}(g_i(C))) - \hat{c}_i(\hat{\theta}(g_i(C)))) ,
\]  
(C2)

\[
P_j(C^*) = V(J) - V(J \setminus j) + \hat{c}_j(\hat{\theta}(g_j(C)))) , \text{ and}
\]  
(C3)

\[
P_j(C) = \sum_{i=1}^{n} [v(\hat{\theta}(g_i(C))) - \hat{c}_i(\hat{\theta}(g_i(C))))] - V(J \setminus j) + \hat{c}_j(\hat{\theta}(g_j(C))))
\]  
(C4)
By substituting terms, we see that Equation C1 can be rewritten as:

\[
P_j(C') + V(J \setminus j) - \hat{c}_j(\hat{\theta}(g_j(C'))) \geq P_j(C) + V(J \setminus j) - \hat{c}_j(\hat{\theta}(g_j(C))) + \hat{c}_j(\hat{\theta}(g_j(C))) - \hat{c}_j(\hat{\theta}(g_j(C)))
\]

and we further note that:

\[
\hat{c}_j(\hat{\theta}(g_j(C'))) = c_j(\hat{\theta}(g_j(C'))), \quad \hat{c}_j(\hat{\theta}(g_j(C))) = c_j(\hat{\theta}(g_j(C)))
\]

\[
P_j(C') + V(J \setminus j) - c_j(\hat{\theta}(g_j(C'))) \geq P_j(C) + V(J \setminus j) - c_j(\hat{\theta}(g_j(C)))
\]

Subtracting \(V(J / j)\) from both side of the inequality in Equation C7 yields the following from the proposition:

\[
P_j(C') - c_j(\hat{\theta}(g_j(C'))) \geq P_j(C) - c_j(\hat{\theta}(g_j(C)))
\]

This completes the proof. \[\Box\]

Appendix D. Proof of Proposition 3 (The Payment Functional Form Proposition)

The payment function in our model is as follows

\[
P_j(\hat{c}_j, \hat{c}_{\neg j}) = V(J) + \sum_{k=\neg j} \hat{c}_k(\hat{\theta}_k) - V(J \setminus j)
\]

\[
= \sum_{i \neq j} [v_i(\hat{\theta}_{a^{-1}(i)}) - \hat{c}_i(\hat{\theta}_{a^{-1}(i)})] + v_j(\hat{\theta}_{a^{-1}(j)}) - V(J \setminus j)
\]

The first and third terms in Equation D1 do not include \(\hat{c}_j(.)\). So we can rewrite Equation D1 as:

\[
P_j(\hat{c}_j, \hat{c}_{\neg j}) = \sum_{i \neq j} [v_i(\hat{\theta}_{a^{-1}(i)}) - \hat{c}_i(\hat{\theta}_{a^{-1}(i)})] + v_j(\hat{\theta}_{a^{-1}(j)}) - h_j(\hat{c}_j)
\]

where \(h_j(\hat{c}_{\neg j})\), which replaces \(V(J / j)\), similarly excludes \(\hat{c}_j(.)\).

Next, suppose there is another form of the payment function \(P'_j(\hat{c}_j, \hat{c}_{\neg j})\) in which the first two terms are same as in Equation D2, but the last term is no longer independent of \(\hat{c}_j(.)\):

\[
P'_j(\hat{c}_j, \hat{c}_{\neg j}) = V(J) + \sum_{k=\neg j} \hat{c}_k(\hat{\theta}_k) + h_j(\hat{c}_j, \hat{c}_{\neg j})
\]

In this expression, \(\hat{c}_{\neg j}\) denotes the cost functions of other suppliers than supplier \(j\).

We wish to show that \(h_j(\hat{c}_j, \hat{c}_{\neg j})\) must be independent of \(\hat{c}_j(.)\) if the social choice function is truthfully-implementable using the dominant strategies. We begin by assuming this is not so. So assume that the social choice function is truthfully-implementable in dominant strategies, but for some \(c_j(.)\), \(\hat{c}_j(.)\) and \(\hat{c}_{\neg j}(.)\), we
Consider two cases as follows,

**Case 1:** \( \Theta^*(\hat{c}_j, \hat{c}_{-j}) = \Theta^*(c_j, \hat{c}_{-j}) \) and \( a^*(\hat{c}_j, \hat{c}_{-j}) = a^*(c_j, \hat{c}_{-j}) \), where \( \Theta^*(\hat{c}_j, \hat{c}_{-j}) \) denotes the optimal quality assignment when supplier \( j \)'s reported cost is \( \hat{c}_j \), while the other suppliers' reports are \( \hat{c}_{-j} \). \( a^*(\hat{c}_j, \hat{c}_{-j}) \) denotes the optimal combinatorial allocation under the same reporting costs. Because the mechanism is incentive-compatible in sense of the application of the dominant strategy, if suppliers report their cost functions truthfully, then we have:

\[
P^*_{ij}(\hat{c}_j, \hat{c}_{-j}) - \hat{c}_j(a^*(\hat{c}_j, \hat{c}_{-j}), \Theta^*(\hat{c}_j, \hat{c}_{-j})) \geq P^*_{ij}(c_j, \hat{c}_{-j}) - \hat{c}_j(a^*(c_j, \hat{c}_{-j}), \Theta^*(c_j, \hat{c}_{-j})) \tag{D5}
\]

\[
P^*_{ij}(c_j, \hat{c}_{-j}) - \hat{c}_j(c_j(a^*(c_j, \hat{c}_{-j}), \Theta^*(c_j, \hat{c}_{-j}))) \geq P^*_{ij}(\hat{c}_j, \hat{c}_{-j}) - \hat{c}_j(c_j(a^*(\hat{c}_j, \hat{c}_{-j}), \Theta^*(\hat{c}_j, \hat{c}_{-j})) \tag{D6}
\]

Since, \( \Theta^*(\hat{c}_j, \hat{c}_{-j}) = \Theta^*(c_j, \hat{c}_{-j}) \) and \( a^*(\hat{c}_j, \hat{c}_{-j}) = a^*(c_j, \hat{c}_{-j}) \), Equations D5 and D6 together imply that

\[
P^*_{ij}(c_j, \hat{c}_{-j}) = P^*_{ij}(\hat{c}_j, \hat{c}_{-j}).\]

Also, based on Equation D3, we have \( h_j(\hat{c}_j, \hat{c}_{-j}) = h_j(c_j, \hat{c}_{-j}) \), which is a contradiction of Equation D4.

**Case 2:** \( \Theta^*(\hat{c}_j, \hat{c}_{-j}) \neq \Theta^*(c_j, \hat{c}_{-j}) \) and \( a^*(\hat{c}_j, \hat{c}_{-j}) \neq a^*(c_j, \hat{c}_{-j}) \), or one of these holds. Without loss of generality, suppose that \( h_j(\hat{c}_j, \hat{c}_{-j}) < h_j(c_j, \hat{c}_{-j}) \). Consider the real cost function:

\[
c_j^\varepsilon(a, \Theta) = \begin{cases} 
\sum_{i \neq j} [v_i(\tilde{\theta}_{a}\hat{\theta}^{-i}(0)) - \hat{c}_i(\tilde{\theta}_{a}\hat{\theta}^{-i}(0))] + \sum_{k \neq j \in (j)} v_{jk}(\tilde{\theta}_{a}\hat{\theta}^{-i}(0)) - \varepsilon & \text{if } a = a^*(\hat{c}_j, \hat{c}_{-j}), \Theta = \Theta^*(\hat{c}_j, \hat{c}_{-j}) \\
\sum_{i \neq j} [v_i(\tilde{\theta}_{a}\hat{\theta}^{-i}(0)) - \hat{c}_i(\tilde{\theta}_{a}\hat{\theta}^{-i}(0))] + \sum_{k \neq j \in (j)} v_{jk}(\tilde{\theta}_{a}\hat{\theta}^{-i}(0)) & \text{if } a = a^*(c_j, \hat{c}_{-j}), \Theta = \Theta^*(c_j, \hat{c}_{-j}) \\
\infty & \text{if } \text{others}
\end{cases}
\tag{D7}
\]

For a sufficiently small value of \( \varepsilon > 0 \), supplier \( j \) with cost function type \( c_j^\varepsilon(\cdot) \) will prefer to falsely report her type when others’ cost function types are \( \hat{c}_{-j}(\cdot) \). To see this, we note that due to Equation D7, \( a^*(\hat{c}_j, \hat{c}_{-j}) \) and \( \Theta^*(\hat{c}_j, \hat{c}_{-j}) \) will maximize the following:

\[
\sum_{i \neq j} [v_i(\tilde{\theta}_{a^*(\cdot)}\hat{\theta}^{-i}(0)) - \hat{c}_i(\tilde{\theta}_{a^*(\cdot)}\hat{\theta}^{-i}(0))] + \sum_{k \neq j \in (j)} v_{jk}(\tilde{\theta}_{a^*(\cdot)}\hat{\theta}^{-i}(0)) - c_j^\varepsilon(a, \Theta) \tag{D8}
\]

Note the social choice goal of social surplus maximization also is reflected in D8: when other suppliers’ reports are \( \hat{c}_{-j}(\cdot) \), the optimal allocations will be the same, whether supplier \( j \) reports \( c_j^\varepsilon(\cdot) \) or \( \hat{c}_j(\cdot) \), so that
\( \Theta^*(\hat{c}_j, \hat{c}_{-j}) = \Theta^*(c_j^*, \hat{c}_{-j}) \) and \( a^*(\hat{c}_j, \hat{c}_{-j}) = a^*(c_j^*, \hat{c}_{-j}) \) will be true. Thus, for truth-telling by supplier \( j \) with type \( c_j^* \) to be a dominant strategy requires the following to be true:

\[
P_j^*(c_j^*, \hat{c}_{-j}) - c_j^*(a^*(c_j^*, \hat{c}_{-j}), \Theta^*(c_j^*, \hat{c}_{-j})) \\
\geq P_j^*(c_j, \hat{c}_{-j}) - c_j^*(a^*(c_j, \hat{c}_{-j}), \Theta^*(c_j, \hat{c}_{-j}))
\]  

(D9)

Substituting \( a^*(\hat{c}_j, \hat{c}_{-j}) \) for \( a^*(c_j^*, \hat{c}_{-j}) \), and \( \Theta^*(\hat{c}_j, \hat{c}_{-j}) \) for \( \Theta^*(c_j^*, \hat{c}_{-j}) \) in Equation D9 yields:

\[
P_j^*(c_j^*, \hat{c}_{-j}) - c_j^*(a^*(\hat{c}_j, \hat{c}_{-j}), \Theta^*(\hat{c}_j, \hat{c}_{-j})) \\
\geq P_j^*(c_j, \hat{c}_{-j}) - c_j^*(a^*(c_j, \hat{c}_{-j}), \Theta^*(c_j, \hat{c}_{-j}))
\]  

(D10)

Further substitution of Equations D3 and D7 into Equation D10 gives:

\[
h_j(c_j^*, \hat{c}_{-j}) + \epsilon \geq h_j(c_j, \hat{c}_{-j})
\]  

(D11)

However by the result we obtained in Case 1, and by \( \Theta^*(\hat{c}_j, \hat{c}_{-j}) = \Theta^*(c_j^*, \hat{c}_{-j}), a^*(\hat{c}_j, \hat{c}_{-j}) = a^*(c_j^*, \hat{c}_{-j}) \), we know that this gives

\[
h_j(c_j^*, \hat{c}_{-j}) = h_j(\hat{c}_j, \hat{c}_{-j})
\]  

(D12)

As a result, Equation D11 can be rewritten as \( h_j(\hat{c}_j, \hat{c}_{-j}) + \epsilon \geq h_j(c_j, \hat{c}_{-j}) \). But, by hypothesis, we asserted that

\[
h_j(\hat{c}_j, \hat{c}_{-j}) < h_j(c_j, \hat{c}_{-j}). \quad \text{So it must be that } h_j(\hat{c}_j, \hat{c}_{-j}) + \epsilon \geq h_j(c_j, \hat{c}_{-j}) \text{ will be violated for a small enough value of } \epsilon > 0. \quad \Box
\]

**Appendix E: Further Analysis of Uniqueness of the Payment Function**

Recall that we wished to explore whether the social choice function (including the payment function and the allocation function) will be truthfully-implementable in dominant strategies only if the payment is a functional function like Equation 4. To begin this analysis, we let \( c_j(f) = \sum_k c_{jk}(f) \). It is common in real world applications to encounter cases in which \( c_j(f) \) can be safely assumed to be twice continuously differentiable, with

\[
\partial^2 c_j(f) / \partial f^2 < 0 \quad \forall f \text{ and } c_j(\cdot). \quad f \text{ is shorthand for } f^*(c_j, c_{-j}) = (a^*(c_j, c_{-j}), \Theta^*(c_j, c_{-j})). \quad \text{The term, } f^*(c_j, c_{-j}), \text{ is the optimal assignment of combinations and quality requirements, when supplier } j \text{'s bid is } c_j, \text{ while the other suppliers bid } c_{-j}. \quad \text{Suppose supplier costs, } c_{jk}, \text{ are drawn from an interval } [\underline{c}_{jk}, \overline{c}_{jk}] \text{ of costs. In this case, if truth-telling about its cost function is a dominant strategy for supplier } j, \text{ then the supplier } j \text{'s first order condition for optimality implies that:}
\[
\frac{\partial c_j(f(c_{j1},c_{j2},...,c_{jk}),c_{-j}))}{\partial f} \frac{\partial f}{\partial c_{jl}}, \quad \forall \ c_{-j} \text{ and } c_{jk}
\]

\[
P_j(c_{j1},c_{j2},...,c_{jk},c_{-j}) = 0, l = 1,2,...,k
\]

\[P_j(c_{j1},c_{j2},...,c_{jk},c_{-j})\] is the payment to supplier \(j\), when her costs are \(\{c_{j1},c_{j2},...,c_{jk}\}\), while others’ costs are \(c_{-j}(.)\). Integrating Equation E1 with respect to the variable \(c_{jl} \in [c_{jl},c_{jl}], l = 1,2,...,k\) suggests that for all profiles of cost types \(\{c_{j1},c_{j2},...,c_{jk},c_{-j}\}\), we will have:

\[
P_j(c_{j1},c_{j2},...,c_{jk},c_{-j}) = P_j(c_{jl},c_{jl},...,c_{jl},c_{-j})
\]

\[
+ \int_{c_{jl}}^{c_{jl}} \frac{\partial c_j(f(c_{j1},c_{j2},...,c_{jk},c_{-j}))}{\partial f} \frac{\partial f}{\partial s} ds
\]

Consider the allocation function \(f^*(c_j,c_{-j}) = (a^*(c_j,c_{-j}), \Theta^*(c_j,c_{-j}))\), which satisfies Equation 4, the payment function. Under our assumption that \(f^*(\cdot)\) must satisfy

\[
\sum_{\alpha} \left( \frac{\partial v_j(f^*(C))}{\partial f} - \frac{\partial c_j(f^*(C))}{\partial f} \right) = 0, \quad \forall \ C = \{c_i(\cdot), i = 1,2,...,n\},
\]

where, \(v_j(f^*(C)) = \sum_{\alpha \in \Theta_j} \theta_{\alpha}\)

Using the implicit function theorem and our assumption about \(c_j(f)\), we see that \(f^*(\cdot)\) has non-zero partial derivatives: \(\frac{\partial f(c_{j1},c_{j2},...,c_{jk},c_{-j})}{\partial c_{jl}} \neq 0 \forall j \text{ and } l\), and is continuously differentiable. We now substitute for

\[
\frac{\partial c_j(f(c_{j1},c_{j2},...,c_{jk},c_{-j}))}{\partial f} \frac{\partial f}{\partial s}
\]

in Equation E2 by using E3, and for any associated bids profile \((c_{j1},c_{j2},...,c_{jk},c_{-j})\), we derive:

\[
P_j(c_{j1},c_{j2},...,c_{jk},c_{-j}) = P_j(c_{jl},c_{jl},...,c_{jl},c_{-j})
\]

\[
+ \int_{c_{jl}}^{c_{jl}} \left( \sum_{\alpha} \frac{\partial v_j(f^*(C))}{\partial f} - \sum_{\alpha \neq j} \frac{\partial c_j(f^*(C))}{\partial f} \right) \frac{\partial f}{\partial s} ds
\]

\[
= P_j(c_{jl},c_{jl},...,c_{jl},c_{-j}) + \left[ \sum_{\alpha \neq j} \frac{\partial v_j(f^*(C))}{\partial f} - \sum_{\alpha \neq j} \frac{\partial c_j(f^*(C))}{\partial f} \right] df
\]

But Equation E4 will be true, if and only if:

\[
P_j(c_{jl},c_{jl},...,c_{jl},c_{-j}) = \sum_{\alpha \neq j} \left[ v_j(f^*(c_{jl},c_{jl},...,c_{jl},c_{-j}) - c_i(f^*(c_{jl},c_{jl},...,c_{jl},c_{-j}))))
\]

\[
+ v_j(f^*(c_{jl},c_{jl},...,c_{jl},c_{-j}))) - h_j(c_{-j})
\]
and Equation E5 is the same as Equation D2. Thus, in this setting, the payment scheme in Equation 4 is the only payment mechanism that satisfies E3, then it is truthfully-implementable in dominant strategies.

**APPENDIX F: AN ILLUSTRATION OF THE MA-VCG MECHANISM**

We provide an illustration of the MA-VCG mechanism based on the procurement of a bundle of goods. The procedure involves several steps leading to what mix of quality and trade items in a bundle maximize value in auction procurement. First, we specify the cost function for quality level of the trade items as they are delivered by the suppliers who make bids. We also specify the utility for quality from the point of view of the buyer—in this case, the auctioneer. Then, associated with the supply bids and the auctioneer’s utility function, we show the search space for the optimal allocation of supplies to the auctioneer.

**Assumptions.** The auctioneer’s goal is to procure three trade items, A, B and C. There are three potential suppliers. Supplier 1 can supply at three trade items, however, Suppliers 2 and 3 cannot individually supply all three. However, together Suppliers 2 and 3 are able to provide the auctioneer with the full bundle. We further assume that there are two levels of quality for all of the trade items to be procured, high and low. The low quality standard (L) is associated with a quality score (\(x\)) of 3, and the high quality standard (H) corresponds to a quality score of 4. The goal of the auctioneer is to maximize the value of quality less procurement cost. (See Tables F1 and F2.)

**Table F1. The Suppliers’ Cost Function for Quality**

<table>
<thead>
<tr>
<th>Suppliers (Bidders)</th>
<th>Trade Items’ Supply Bundles with Costs Assigned</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ABC</td>
</tr>
<tr>
<td>1</td>
<td>{3x,3x,4x}</td>
</tr>
<tr>
<td>2</td>
<td>#</td>
</tr>
<tr>
<td>3</td>
<td>#</td>
</tr>
</tbody>
</table>

Note: # denotes that the available information for the auctioneer/buyer (i.e., the “report”) does not include the corresponding bundles. \(x\) is the quality score, which is 3 for low quality and 4 for high quality.

**Table F2. The Auctioneer’s Utility Function for Quality**

<table>
<thead>
<tr>
<th>Trade Items</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Utility</td>
<td>4x</td>
<td>6x</td>
<td>5x</td>
</tr>
</tbody>
</table>

Note: \(x\) is the quality score, which is 3 for low quality and 4 for high quality.

**Search Procedure for the Optimal Allocation of Suppliers.** The search procedure involves the enumeration of all of the different combinations of quality bundles—which reflect acceptable combinations of high and low quality trade items—to determine the highest net value based on total utility less total cost. (See Table F3.)

Based on the information presented in Table F3, we can see that there are two optimal allocations to suppliers, Allocation 9 and Allocation 13, each with a net value of 24 for the auction. The auctioneer should be indifferent between these two allocations in value terms, but may recognize that the quality of the Allocation 9 dominates the quality of Allocation 13 due to the higher quality level for Item A in Allocation 9. Other considerations may apply too.

**The Payment to Winning Suppliers.** We now consider the payment function for transfers from the auctioneer to the suppliers. According to the base case MA-VCG mechanism that we presented in this article, we need to examine Allocation 9 and Allocation 13 separately to determine what Suppliers 2 and 3 will earn. We remind the reader that

\[ P(\hat{c},J) = V(J) + \hat{c}(\hat{\theta}_{\pi(J)} - V(J \setminus J) \text{, where the first term, } V(J), \text{ is the maximum social surplus that can be achieved by having all suppliers' participate in the auction. The second term, } \hat{c}(\hat{\theta}_{\pi(J)}), \text{ is the cost for the supplier associated with} \]


providing the corresponding goods to the auctioneer. The third term, \( V(J \setminus j) \), is the maximum social surplus that can be achieved in the auction without supplier \( j \)'s participation. So supplier \( j \)'s payment includes the difference between the maximum achievable social surplus with or without her participation, plus the cost of her supply of the winning goods to the auctioneer that meet the quality requirements. If the auctioneer chooses Allocation 9, \{H,H,H\}, then the payments to the suppliers are:

- Supplier 2: \( 24 + 24 - 20 = 28 \)
- Supplier 3: \( 12 + 24 - 20 = 16 \)

If the auctioneer chooses Allocation 13, \{L,H,H\}, then the payments to suppliers will be:

- Supplier 2: \( 20 + 4 = 24 \)
- Supplier 3: \( 12 + 4 = 16 \)

| Table F3. Evaluation of the Procurement Combinatorial Auction Allocations |
|---------------------------------|-----------|---------|---------|---------|
| **ALLOCATION** | **QUALITY** | **UTILITY** | **COST** | **NET VALUE** |
| Supplier 1 Selected for Procurement | | | | |
| 1 | \{H,H,H\} | 60 | 40 | **20** |
| 2 | \{H,H,L\} | 55 | 36 | 19 |
| 3 | \{H,L,H\} | 54 | 37 | 17 |
| 4 | \{H,L,L\} | 49 | 33 | 16 |
| 5 | \{L,H,H\} | 56 | 37 | 19 |
| 6 | \{L,H,L\} | 51 | 33 | 18 |
| 7 | \{L,L,H\} | 50 | 34 | 16 |
| 8 | \{L,L,L\} | 45 | 30 | 15 |
| Suppliers 2 and 3 Selected for Procurement | | | | |
| 9 | \{H,H,H\} | 60 | 36 | **24** |
| 10 | \{H,H,L\} | 55 | 33 | 22 |
| 11 | \{H,L,H\} | 54 | 34 | 20 |
| 12 | \{H,L,L\} | 49 | 31 | 18 |
| 13 | \{L,H,H\} | 56 | 32 | **24** |
| 14 | \{L,H,L\} | 51 | 29 | 22 |
| 15 | \{L,L,H\} | 50 | 30 | 20 |
| 16 | \{L,L,L\} | 45 | 27 | 18 |

**Note:** The allocation algorithm is as follows: (1) For supplier combinations that match the auctioneer’s requirements, compute total utility, total cost and net value. (2) Identify the highest net value allocation for each supplier combination. (3) Select supplier allocation to maximize net value. (4) If ties exist, apply other preferences to break ties. The asterisks are: ** = Supplier 1 can meet auctioneer’s requirements alone, whose maximum net value is 20; and *** = Suppliers 2 and 3 can jointly meet the requirements, whose maximum net value is 24.

The overall surplus of the auctioneer according to MA-VCG is determined based on the two optimal allocations. It is computed based on the corresponding utility of the bundle to auctioneer, less the sum of payments to Suppliers 2 and 3. Thus, the auctioneer’s surplus for Allocation 9 is given by \( 60 - 28 - 16 = 16 \), while that for Allocation 13 is given by \( 56 - 24 - 16 = 16 \).

**The Revised Mechanism and Its Payment Function.** In the revised MA-VCG mechanism, the allocation and corresponding quality standard are the same as in the original base version of the mechanism. The payment to the winners in revised mechanism is now \( P_j(\hat{\theta}, \hat{\theta}, \bar{\theta}^*(\alpha_j)) = V(J) + \hat{\theta}_j(\bar{\theta}^*(\alpha_j)) - V(J_{-j}, \bar{\theta}_j) \). The reader should specifically note that the third term is different, and supplier \( j \) has cost function \( \bar{\theta}_j(\cdot) \). Considering the definition of \( \bar{\theta}_j(\cdot) \), then
\( V(J_{-j}, \overline{c}_j) \) is the least maximal social surplus with supplier \( j \)'s participation, given the other suppliers' bids. We also know that \( V(J_{-j}, \overline{c}_j) \geq V(J \setminus j) \) \( \forall j \) is true. For this example, we will further assume a minimal decrease of 1 unit for the value of social surplus and in the payment scheme. Therefore, we apply the cost function \( \{4x, 2.75x\} \) for items AB as \( \overline{c}_j(\cdot) \) for Supplier 2, and the cost function \( \{3.75x\} \) for item C as \( \overline{c}_j(\cdot) \) for Supplier 3. In this way, we derived the revised payments for the winning suppliers in this auction as follows for Allocations 9 and 13:

- If the auctioneer chooses Allocation 9, \( \{H,H,H\} \), then the revised payments are:
  - Supplier 2: \( 24 + 24 - 21 = 27 \)
  - Supplier 3: \( 12 + 24 - 21 = 15 \)

- If the auctioneer chooses Allocation 13, \( \{L,H,H\} \), then the revised payments will be:
  - Supplier 2: \( 20 + 3 = 23 \)
  - Supplier 3: \( 12 + 4 = 16 \)

The surplus of the auctioneer according to the revised MA-VCG mechanism is again determined based on the two optimal allocations. It is computed via the utility of the bundle to the auctioneer, less the sum of payments to Suppliers 2 and 3. Thus, the auctioneer’s surplus for Allocation 9 is given by \( 60 - 27 - 15 = 18 \), while that for Allocation 13 is given by \( 56 - 23 - 15 = 18 \). In both cases, the auctioneer’s surplus is higher for the MA-VCG with the revised payment mechanism than for the base payment scheme, whose surpluses for the auctioneere were 16 for Allocations 9 and 13.