Recommender System Rethink: Implications for an Electronic Marketplace with Competing Manufacturers

Lusi Li, Jianqing Chen, Srinivasan Raghunathan

The University of Texas at Dallas

lusi.li@utdallas.edu, chenjq@utdallas.edu, sraghu@utdallas.edu

Abstract

Recommender systems that inform consumers about their likely ideal product have become the cornerstone of eCommerce platforms that sell products from competing manufacturers. Using a model of an electronic marketplace in which two competing manufacturers produce substitutable products and sell the products through a common retail platform, we study the effect of recommender systems on the marketplace, manufacturers, and consumers. Consumers are heterogeneous with respect to their preference for and awareness about the two products. The recommender system recommends the product that offers a higher expected consumer net utility. Our analysis shows that the recommender system may benefit or hurt the retailer and the manufacturers, depending on the tradeoff between two opposing effects of recommender systems. The *demand effect* of the recommender system increases the proportion of consumers that are aware of at least one of the products, thus increasing the overall market size and benefiting the sellers. The *substitution effect* of the recommender system causes the demand function to be more price elastic through two forces—one that shifts some consumers that are aware of a single product to ones that are aware of both products, thus making them comparison shoppers, and another that induces manufacturers to use price to gain informational advantage among consumers that are unaware of either product. The magnitudes of these two effects depend on the relative sizes of consumer segments with different awareness levels and the recommender system precision. An increase in precision mitigates the substitution effect of the recommender system. However, even perfect precision cannot guarantee that the recommender system benefits the retailer and manufacturers. On the other hand, regardless of the precision, the recommender system benefits the consumers and social welfare. We further show that by strategically committing to not recommend any product to some consumers—those whose preference for a product is distorted by prices—, the retailer can mitigate the substitution effect and benefit more or lose less from the recommender system.

1 Introduction

Recommender systems have become the hallmark of modern eCommerce platforms. These systems are touted as sales support tools that help consumers identify their "ideal" product from among the vast variety sold by an eCommerce platform (Hennig-Thurau et al., 2012). It has been reported that over 35% of sales on Amazon.com and more than 60% of the rentals on Netflix result from recommendations (Fleder et al., 2010).

Commercial recommender systems vary with respect to the algorithms used to generate recommendations, the timing related to when recommendations are presented to a consumer, the way recommendations are presented, and the type of products—complementary or competing—recommended. For instance, while content-based recommender systems use product information (e.g., genre, mood, and author in the case of books) to recommend items that are similar to products that a target consumer previously bought or liked, collaborative filter-based recommender systems recommend products based on purchase history or taste of similar consumers. On the timing dimension, while some systems recommend complementary products after a consumer made a purchase, others recommend products before the purchase, viz., immediately after the consumer logs into the system or when the consumer searches for a product. Despite these differences, the main goal of most recommender systems seems to be to introduce consumers to new products and help them select desirable products from among myriad choices (Resnick and Varian, 1997).

Research indicates that a recommender system affects consumer decision making (Tam and Ho, 2006). For example, recommender systems may inform consumers about products which they may not be aware of ("informative role") and thus increase the consumers' consideration set, and when a consumer is already aware of the recommended product, the recommendation may increase the purchase probability of that product ("persuasive role") (Gretzel and Fesenmaier, 2006; Tam and Ho, 2005; Fleder and Hosanagar, 2009). Research has also examined the impact of recommender systems on product sales. For instance, some studies argue that recommender systems contribute to the long tail effect by exposing consumers to niche products, but others argue that some recommender system designs make the already popular product even more popular (Mooney and Roy, 2000; Fleder and Hosanagar, 2009). Departing from these studies, in this paper we aim to analytically study how recommender systems affect the upstream competition between manufacturers that sell competing products through an eCommerce platform that provides recommendations.

The question of how recommender systems affect the price competition between substitutable products is important to both practitioners and academics. The question becomes especially important in a context of a dominant eCommerce platform that sells competing products from different manufacturers while simultaneously recommending a sub set of these products, because such a two level channel structure with one dominant eCommerce platform is commonly observed in practice (e.g., Amazon's online marketplace). In such contexts, although each manufacturer views the substitutable products from other manufacturers as competitors, the platform may view the products as satisfying the different needs of different consumers. Therefore, from a platform's perspective, an analysis of recommender systems' effects on both consumers (demand side) and manufacturers (supply side) is essential for a more complete understanding of the implications of recommender systems. However, the effect of recommender systems on the manufacturers remains unexplored and unclear despite the ubiquitous presence of these systems in eCommerce platforms.

In order to address this question, we develop an analytical model in which two manufacturers produce and sell two substitutable products through a common retail platform (hereafter referred to as the retailer). Manufacturers set the sales prices and the retailer charges a fixed commission that is a percentage of the sales price. Consumers that visit the retailer have heterogeneous preferences. Furthermore, consumers are heterogeneous in their awareness about the two products. We distinguish three types of consumers based on their awareness about products when they visit the retailer: partially-informed consumers who are aware of only one product, fully-informed consumers who are aware of both products, and uninformed consumers who are aware of neither product. The recommender system is designed to help consumers find the product that offers the maximum value to them; that is, based on the information it has about the consumer, the recommender system recommends the product that would provide a higher expected net utility to the consumer.

Our analysis shows that the recommender system may benefit or hurt the retailer and the manufacturers depending on the trade off between two opposing effects. The positive effect of the recommender system is that it increases the proportion of consumers that are aware of at least one of the products, thus increasing the overall market size. The negative effect is that the recommender system converts some consumers that are initially aware of a single product to ones that are aware of both products. These consumers become comparison shoppers as a result of the recommendation and some may end up substituting the product they were initially aware of with the recommended one. This effect increases the price competition between the two manufacturers. We call the former the *demand effect* and the latter the *substitution effect*. The magnitudes of these two effects depend on the relative sizes of the three consumer segments and the recommender system precision.

An increase in precision mitigates the substitution effect of recommender systems. Intuitively, an increase in precision causes the recommender system to assign a greater weight to a consumer's location (or signal about the location) relative to the product prices while generating a recommendation. Therefore, an increase in precision effectively reduces the marginal benefit from and hence the sellers' incentive for a price decrease and thus softens the price competition. However, even perfect precision cannot guarantee that the recommender system benefits the retailer and manufacturers. While the sellers may not benefit from the recommender system, consumers and social welfare always benefit from it: consumers benefit from a larger market size, lower prices, and smaller misfit costs, and social welfare benefits from a larger market size and smaller misfit costs.

We further show that by strategically committing to not recommend any product to some consumers—-whose preference for a product based on net utility conflicts with that based on product fit—, the retailer can mitigate the intensification of price competition and benefit more or lose less from the recommender system.

1.1 Related Literature

The existing literature on recommender systems generally falls into three streams. The first stream focuses on recommendation algorithm design. Much of the work in this stream has emphasized predicting consumer preference and recommending products that match consumer preference. Adomavicius and Tuzhilin (2005) provide a review of this work. Algorithms to increase recommendation diversity have also been proposed (McNee et al., 2006; Vargas and Castells, 2011; Pu et al., 2011). Other recommender systems maximize seller's profit by incorporating factors such as profit margin and product inventory (Chen et al., 2008; Bodapati, 2008). Different from this literature stream, we take the recommendation algorithm as exogenously given and focus on the economic impact of recommender systems on sellers. In particular, we assume that the recommender system recommends the product that offers the maximum consumer surplus, which is consistent with the broad literature on recommendation algorithms.[1]

The second stream of literature focuses on the impact of recommender systems on consumers' decision making and choices. Senecal and Nantel (2004) show that recommended products are selected twice as often as non-recommended products. This influence is moderated by the type of

[1] http://www.digitalbookworld.com/2012/consumers-like-and-trust-amazon-book-recommendations-despite-industry-jitters/ provides an insider look at recommendation algorithms used by Amazon.com. According to former Amazon engineers who were involved in the design and development of Amazon's recommendation engines, it is unlikely that Amazon's recommendation engines are set up to do anything other than recommend products that users are most likely to buy—contrary to the rumors about whether it is used for marketing purposes. They stated, "Anything that doesn't maximize the usefulness to customers of the recommendations will hurt sales, and it is hard to make up for that huge cost any other way."

recommendation source and the type of product. Cooke et al. (2002) find that context, familiarity, and additional information can affect consumer's reaction to recommendations. Consistent with empirical findings in this stream of research, we develop a model in which recommended products have a higher average chance of being bought than non-recommended products, but whether the recommended product is purchased depends on consumer preference, price, and awareness about the competing product.

The third stream of research has examined the effect of recommender systems on product sales and sales diversity. Using a simulation model, Fleder and Hosanagar (2009) study the effect of recommender system on the diversity of sales and show that recommendations made on the basis of sales and ratings reinforce the popularity of already popular products. Thus, they find that the aggregate diversity could decrease although the individual-level diversity increases. Fleder et al. (2010) empirically show that recommender systems can lead to consumers purchasing more similar items. Using a simulation model built based on real purchase data of a video-on-demand retailer, Hinz and Eckert (2010) show how different classes of search and recommendation tools affect the distribution of sales across products, total sales, and consumer surplus. Brynjolfsson et al. (2011) find that a firm's online sales channel has a slightly higher diversity than its offline channel. They suggest recommender systems as a possible driving force for the difference in the two channels, but do not isolate the specific effect of recommender system. Oestreicher-Singer and Sundararajan (2012a) empirically show that recommender system can lead to a redistribution of demand. Their results indicate that a recommendation network induces a significantly flatter demand and revenue distributions. Oestreicher-Singer and Sundararajan (2012b) show that on average the explicit visibility of a co-purchase relationship can amplify the influence that complementary products have on each other's demands. Pathak et al. (2010) show that the strength of recommendations has a positive effect on sales and prices and that this effect is moderated by the recency effect. Jabr and Zheng (2013) analyze the effect of recommendations and word-of-moth reviews on product sales in a competitive environment and show that higher referral centrality of competing products is associated with lower product sales. In contrast to this stream of research, we examine the impact of recommender systems on competing products using an analytical model and study upstream effects of downstream deployment of a recommender system.

Research that has examined the impact of recommender systems using an analytical model is limited. Hervas-Drane (2009) show that when recommender systems based on consumer taste are introduced alongside traditional word-of-mouth, there is a positive impact on consumers interested in niche products and a decrease in market concentration. Bergemann and Ozmen (2006) use a two-stage game to show how a firm can strategically choose its price in the first stage to generate recommendations in the second stage. Our study is different from these in that we examine the effect of recommender systems in the setting of competing sellers in a channel structure, while previous studies consider a single seller and ignore the strategic interactions between sellers.

Our study is also related to targeted advertising literature in the sense that recommender systems also expose consumers to new products as advertising does. Several studies examine how firms choose target strategy and how targeted advertising affects firms' profits compared to random advertising (Iyer et al., 2005; Esteban and Hernandez, 2007; Gal-Or and Gal-Or, 2005).Our study differs from those in the targeted advertising literature in the following ways. In the targeted advertising models, manufacturers are the decision makers that choose the strength of targeted advertising as well as product prices. In contrast, in our model recommendations are chosen by the common retail platform but the manufacturers set prices. In the targeted advertising literature, manufacturers choose advertising to maximize their own payoff, but the goal of recommendation in our model is to maximize the expected consumer surplus. While pricing decisions are made after observing the advertising intensity in the targeted advertising models, manufacturers choose the prices before the recommendation is delivered in our context. In summary, our model seeks to specifically analyze the recommender system context.

2 Model

We consider a two-level channel structure with two manufacturers (A and B), one common retailer (R), and a continuum of consumers with heterogeneous preferences. Manufacturer A(B) produces product A(B) and sells the product via R. Each manufacturer sets the price of its product, and the retailer charges the manufacturers a commission equal to α fraction of the price on each sale.[2] The two products are horizontally differentiated and have different levels of misfit to different consumers. In particular, we assume that the products are located at the two end points of a Hotelling line of a unit length, with product A being at 0 and product B being at 1. Consumers are uniformly distributed along the line. The distance between a product and a consumer measures the degree

[2] An alternative setting is one in which R buys from manufacturers and resells them to consumers. Amazon.com uses such a wholesale pricing scheme for some products and the platform scheme we consider in our paper for other products. It is noteworthy that Amazon.com sells 93% of products in the electronics category, 83.3% of Shoes, 96.9% of products in Sports & Outdoors category, and 96.8% of products in the Jewelry category using the platform scheme (Jiang et al., 2011).

of misfit of the product to the consumer. The misfit cost is the degree of misfit times a unit misfit cost t. A consumer's utility for product i, $i \in \{A, B\}$, is equal to the value of the products v net the misfit cost and product price p_i. For a consumer located at z, the net utility from buying product A is $U_A = v - zt - p_A$ and from buying product B is $U_B = v - (1-z)t - p_B$.

We distinguish three types of consumers in terms of their awareness about the two products: fully-informed consumers, uninformed consumers, and partially-informed consumers. Fully-informed consumers are aware of both products. Uninformed consumers are aware of neither product. Partially-informed consumers include consumers who are only aware of product A and consumers who are only aware of product B. Without loss of generality, we assume that the number of consumers is a unit mass. The size of the informed consumer segment is θ_b and the size of each group of the partially-informed consumer segment is θ. Thus, the proportion of uninformed consumers is $1 - 2\theta - \theta_b$. This basic awareness structure is implied by the existence of advertising sources that can be employed by manufacturers such as TV, newspapers, and Internet advertising. The awareness of a consumer is independent of her location. We refer to the differentiation of the firms along the Hotelling line as *location differentiation* and that along the consumer awareness dimension as *informational differentiation*.

Each consumer has a unit demand and can only purchase a product that she is aware of. Consistent with the horizontal differentiation literature (Tirole, 1988), we assume that the market is fully covered for consumers who are aware of at least one product. The full market coverage assumption holds when v is high relative to price and misfit cost.[3] Therefore, if a consumer is aware of only one product, she would purchase it. If a consumer is aware of both products, she purchases the product which offers a higher net utility. We denote as z_0 the location of the marginal consumer who would be indifferent between these two products if she were fully informed. Based on the utility function, we have

$$z_0 = \frac{p_B - p_A + t}{2t} \tag{1}$$

Consumers located at $z < z_0$ would, if fully informed, find product A to be their first choice. On the contrary, consumers located at $z > z_0$ would find product B to be their first choice.

The primary goal of the recommender system is to help consumers make better decisions, which, as mentioned earlier, is the stated goal of the well-known Amazon's recommender system. To capture this feature, we assume that the recommender system would recommend the product that offers a higher expected net utility to a consumer based on the information it has about the consumer.

[3] We provide the precise mathematical condition as a technical assumption later in this section.

Recommender systems use information such as purchase data, rating data, and profile data to estimate consumer's preference which is modeled as consumer's location on the Hotelling line. The recommender system may be imperfect in estimating a consumer's preference and we use a commonly used approach to model this estimation (e.g., Lewis and Sappington 1994; Johnson and Myatt 2006). In particular, the retailer observes a signal s regarding consumer's location. The signal equals the consumer's true location with probability β, and with probability $1-\beta$ the signal is uninformative and follows the prior distribution of consumer location. That is, $P(s=y|z=y)=\beta$ and $P(s\neq y|z=y)=1-\beta$, where $y\in[0,1]$. The model indicates that the signal is informative (i.e, provides useful information for the retailer to estimate the consumer's preference) but noisy (i.e, does not perfectly reveal the true preference). We refer to β as the precision of the recommender system.

The sequence of events is as follows. In stage 1, manufacturers set prices p_A and p_B simultaneously. In stage 2, consumers visit the retailer and make their purchase decisions. Two scenarios are considered: one without the recommender system and the other with a recommender system. We use the scenario without the recommender system as the benchmark to analyze the effect of the recommender system. In the scenario without the recommender system, consumers make their purchase decisions based on their awareness about products. In the scenario with the recommender system, the retailer recommends one product to each consumer,[4] and consumers make their purchase decisions with this additional information.

As the game sequence implies, the recommender system provides the recommendation before a consumer makes the purchase decision. Thus, the model is appropriate for contexts where consumers receive recommendations when they log in or when they search for a product to buy. For instance, Amazon.com's personalized recommendations when consumers log in and the recommendation feature "what items do consumers buy after viewing this item?" fit our model.

We assume without loss of generality that the fixed and marginal production costs are zero. Further, we assume that the cost of developing the recommender system and the cost of providing a recommendation are zero. A consumer's own awareness and her preference are private information. All other model parameters are common knowledge. All players are risk neutral.

For our analysis, we focus on non-trivial cases where the competition between manufacturers plays a role in the equilibrium, and we make the following technical assumptions.

Assumption 1: $\frac{2(\theta+\theta_b)t}{\theta_b}<v<\frac{(2\theta+\theta_b)^2t}{2\theta\theta_b}+t$

[4] In Section 4, we propose and evaluate a case in which the recommender system does not always recommend a product to a consumer.

Assumption 2: $\beta > \frac{1}{9}$

The lower bound on v in Assumption 1 ensures that the market is fully covered, and the upper bound ensures that manufacturers do not have an incentive to serve only the partially informed consumers who are aware of their own product and ignore the fully informed consumers. That is, the two firms compete for the common demand that consists of fully informed consumers. Analogously, the lower bound on β in Assumption 2 ensures that the recommender system does provide a fairly good estimation such that firms factor in the effect of the recommender system in their pricing decisions.

In the base model, we assume that the recommender system is exogenously specified. The retailer has a passive role in the sense that it does not strategically choose a recommendation policy that maximizes its own payoff. We keep the base model parsimonious in order to focus on the effect of recommender systems commonly employed by downstream retailers on the upstream competition in a channel structure. In Section 4, we consider a case where the retailer adopts a recommendation policy that allows it to not provide a recommendation for some consumers, while still maximizing the expected surplus for others.

3 Impact of Recommendation

In this section, based on backward induction, we first derive subgame perfect equilibria for the case without the recommender system and the case with the recommender system. Then, we analyze the effect of the recommender system by comparing the equilibria in the two cases.

3.1 Benchmark Case (No Recommendation)

Without the recommender system, a consumer who is only aware of product i would purchase product i, $i \in \{A, B\}$. A consumer who is aware of both products would purchase the one that yields a higher net utility. A consumer who is aware of neither product would not buy any products. Therefore, the demand functions for the two products can be formulated as:

$$\begin{aligned} D_A &= \theta + z_0\theta_b \\ D_B &= \theta + (1 - z_0)\theta_b \end{aligned} \tag{2}$$

The manufacturers maximize their profits by choosing their optimal prices:

$$\max_{p_i} \pi_i = (1-\alpha)p_i D_i \tag{3}$$

Based on their best response to each other, we obtain the equilibrium price and demand for each manufacturer. The following lemma summarizes the equilibrium outcome.

Lemma 1. *In the absence of the recommender system, the equilibrium prices, demands, manufacturer profits, retailer profit, consumer surplus, and social welfare are as follows:*

(a) *Price:*

$$p_A^* = p_B^* = t + \frac{2t\theta}{\theta_b} \tag{4}$$

(b) *Demand:*

$$D_A^* = D_B^* = \theta + \frac{\theta_b}{2} \tag{5}$$

(c) *Manufacturer profit:*

$$\pi_A^* = \pi_B^* = \frac{(1-\alpha)(2\theta+\theta_b)^2 t}{2\theta_b} \tag{6}$$

(d) *Retailer profit:*

$$\pi_R^* = \frac{\alpha(2\theta+\theta_b)^2 t}{\theta_b} \tag{7}$$

(e) *Consumer surplus:*

$$CS = (2\theta+\theta_b)\left[v - \frac{(2\theta+\theta_b)t}{\theta_b}\right] - \frac{(4\theta+\theta_b)t}{4} \tag{8}$$

(f) *Social welfare:*

$$W = (2\theta+\theta_b)v - \frac{(4\theta+\theta_b)t}{4} \tag{9}$$

Proof. All proofs are in the appendix unless indicated otherwise. □

The price expression in Lemma 1(a) has a simple interpretation. The first term is the price under full information. The second term is the markup manufacturers can charge because of the monopoly turf (i.e., the partially informed segment) they have and is due to the lower elasticity of demand associated with informational differentiation under partial information compared to full information when there is no informational differentiation between the two firms. The equilibrium price is increasing in θ but decreasing in θ_b. Intuitively, any increase in the size of the monopoly turf increases the markup manufacturers can charge. On the other hand, θ_b is the size of the common turf manufacturers compete in. Any increase in the size of this group intensifies the price competition. It is also intuitive that an increase in either θ or θ_b increases the demand for both manufacturers because the total market size increases in this case.

3.2 With Recommender System

When the recommender system is in place, each consumer is recommended the product which the retailer believes delivers a higher expected net utility to the consumer. Based on the way we model the imperfect signal, using Bayesian updating, we can derive the retailer's expected location for a consumer when the signal is y to be:

$$E(z|s=y) = \beta y + \frac{1-\beta}{2} \tag{10}$$

(Proof is provided in the appendix). We denote y_0 as the marginal signal under which the expected net utilities from buying A and from buying B are equal. If the recommender system receives a signal less than y_0, the retailer recommends product A; otherwise, it recommends product B. Based on the expected location in Equation (10), we can derive the relationship between the marginal signal y_0 and marginal consumer z_0 defined in Equation (1) as follows:

$$y_0 = \frac{1}{\beta}(z_0 - \frac{1-\beta}{2}) \tag{11}$$

We next formulate the demand function by analyzing each consumer segment. Without loss of generality we assume $p_A \leq p_B$, which implies $z_0 \geq \frac{1}{2}$ and $y_0 \geq z_0$. We first consider the segment in which consumers are only aware of product A. Within this segment, a consumer whose location is less than z_0 buys product A regardless of the recommendation she receives. A consumer whose location is between z_0 and 1 buys A if she is recommended A because she will be aware of only A. If she is recommended B and so is aware of both products, she buys product B by the definition of marginal consumer. A consumer located between z_0 and y_0 receives recommendation for product A (and thus buys product A) with probability $(1-\beta)y_0 + \beta$. A consumer located between y_0 and 1 receives recommendation for product A (and thus buys product A) with probability $(1-\beta)y_0$. All together, the demand for A from consumers who are only aware of product A is

$$d_A = \theta\Big[z_0 + \int_{z_0}^{y_0} P(s \leq y_0|z)dz + \int_{y_0}^{1} P(s \leq y_0|z)dz\Big] = \theta\left[y_0 + z_0(1-y_0)(1-\beta)\right]$$

and the demand for product B is $d_B = \theta - d_A$.

Using a similar logic, we can derive the demand functions for other consumer segments. Among consumers who are only aware of product B, the demand for product A is

$$d_A = \theta \int_0^{z_0} P(s \leq y_0|z)dz = \theta z_0 \Big[(1-\beta)y_0 + \beta\Big]$$

and the demand for product B is $d_B = \theta - d_A$. Among consumers who are aware of both products A and B, the demand for product A is $d_A = \theta_b z_0$ and the demand for B is $d_B = \theta_b - d_A$. Among consumers who are aware of neither product, if the signal is less than y_0, a consumer is recommended product A and purchases A. Therefore, the demand for product A is $d_A = (1 - 2\theta - \theta_b)y_0$ and the demand for B is $d_B = (1 - 2\theta - \theta_b) - d_A$ for the fully informed segment.

Aggregating the demands from each consumer segment, we can derive the total demand for product A as $D_A = (1-\theta-\theta_b)y_0 + (\theta+\theta_b)z_0$. Substituting the values for z_0 and y_0 from Equations (1) and (11), we can derive the demand functions for products A and B as follows:

$$\begin{aligned} D_A &= \tfrac{1}{2} + \tfrac{1-(1-\beta)(\theta+\theta_b)}{2t\beta}\Big(p_B - p_A\Big) \\ D_B &= \tfrac{1}{2} + \tfrac{1-(1-\beta)(\theta+\theta_b)}{2t\beta}\Big(p_A - p_B\Big) \end{aligned} \tag{12}$$

Similar to the benchmark case, we can formulate the manufacturers' optimization problems. Based on their best response to each other, we obtain the equilibrium price and demand for each manufacturer. The following lemma summarizes the equilibrium outcomes.

Lemma 2. *When the retailer uses the recommender system, the equilibrium prices, demands, manufacturer profits, retailer profit, consumer surplus, and social welfare are as follows:*

(a) *Price:*

$$p_A^* = p_B^* = \frac{t\beta}{1-(1-\beta)(\theta+\theta_b)} \tag{13}$$

(b) *Demand:*

$$D_A^* = D_B^* = \frac{1}{2} \tag{14}$$

(c) *Manufacturer profit:*

$$\pi_A^* = \pi_B^* = \frac{t\beta(1-\alpha)}{2-2(1-\beta)(\theta+\theta_b)} \tag{15}$$

(d) *Retailer profit:*

$$\pi_R^* = \frac{t\beta\alpha}{1-(1-\beta)(\theta+\theta_b)} \tag{16}$$

(e) *Consumer surplus:*

$$CS = v - \frac{t\beta}{1-(1-\beta)(\theta+\theta_b)} - \frac{t}{4} - \frac{(1-\theta_b-\theta)(1-\beta)t}{4} \tag{17}$$

(f) *Social welfare:*

$$W = v - \frac{t}{4} - \frac{(1-\theta_b-\theta)(1-\beta)t}{4} \tag{18}$$

The price expression given in Lemma 2(a) enables easier interpretation when written as $t - \frac{(1-\beta)(1-\theta-\theta_b)t}{1-(1-\beta)(\theta+\theta_b)}$. As in the price expression for the benchmark case, the first term here is the price under full information. The second term is the discount manufacturers offer to enable more recommendations in their favor and hence enlarge their monopoly turf; it reflects the fact that the demand is more elastic when informational differentiation is influenced by price as is the case with the presence of the recommender system that uses consumer net utility in the recommendation strategy. Lemma 2(a) shows that the recommender system precision plays a role in the price competition. As in the case with no recommender system, an increase in θ softens price competition and increases price. Surprisingly, in contrast to the result in the no recommender system case, an increase in θ_b softens the price competition when the recommender system is present. An increase in θ_b has two effects. On the one hand, increasing θ_b shrinks the monopoly turf (In the equilibrium, the partially-informed segment is $1-\theta-\theta_b$, the fully-informed segment is $\theta+\theta_b$), thus potentially intensifying the competition. On the other hand, increasing θ_b reduces the influence of recommender system since fully-informed consumers are not affected by recommendation, thus manufacturers have less incentive to compete in price in order to be recommended by retailer. The second effect offsets the first, and hence increasing θ_b softens the competition between manufacturers. Lemma 2(b) shows that the entire market is covered when the recommender system is in place, regardless of precision. This follows from the observation the recommender system provides a recommendation to every consumer and therefore every consumer is aware of at least one product and buys.

3.3 Impact of Recommender System

We now can assert the effect of recommender systems by comparing equilibrium quantities in the scenario without the recommender system and the scenario with the recommender system. For the comparison purpose, we use the regular notations (e.g., p_i) for the scenario without the recommender system and use the notations with hats (e.g., $\hat{p}_i$) for the scenario with the recommender system.

Proposition 1. *Compared to the scenario without the recommender system, in the presence of the recommender system:*

(a) *Each product's demand is higher; that is,* $\hat{D}_i \geq D_i$*,* $i \in \{A, B\}$*.*

(b) *Each product's price is lower; that is,* $\hat{p}_i \leq p_i$*,* $i \in \{A, B\}$*.*

(c) *The retailer and manufacturers are worse off (i.e.,* $\hat{\pi}_i < \pi_i$*) if* $\theta > \frac{\sqrt{\theta_b}}{2}\left(1 - \sqrt{\theta_b}\right)$ *regardless of the recommender system precision; the retailer and manufacturers are better off if and only if* $\theta \leq \frac{\sqrt{\theta_b}}{2}\left(1 - \sqrt{\theta_b}\right)$ *and* $\beta \geq \max[\frac{(2\theta+\theta_b)^2(1-\theta-\theta_b)}{\theta_b-(2\theta+\theta_b)^2(\theta+\theta_b)}, \frac{1}{9}]$*.*

(d) *Consumer surplus is higher; that is,* $\hat{CS} \geq CS$*.*

(e) *Social welfare is higher; that is,* $\hat{W} \geq W$*.*

The impact of the recommender system can be explained in terms of how the recommender system alters the shape of the demand function for each product, as illustrated by Figure 1. Figure 1 shows the demand of product i as a function of the difference in the two prices, $p_A - p_B$, in both the case without the recommender system and the case with the recommender system. We note that the intercept of the demand function increases from $\frac{2\theta+\theta_b}{2}$ in the no-recommender system scenario to $\frac{1}{2}$ in the recommender system scenario. We refer to this market expansion as the *demand effect* of the recommender system. This is the direct effect of the recommender system and it is positive for the manufacturers and the retailer. On the other hand, there is also a strategic effect: Figure 1 shows the demand becomes more elastic in the presence of the recommender system as indicated by the larger slope of the demand function in the case with the recommender system compared to the case without. We refer to the change in the slope of the demand function as the *substitution effect* of the recommender system.

There are two drivers of the substitution effect. One, the recommender system converts some partially informed consumers (i.e., monopoly turf) to fully informed consumers (i.e., common turf), making them comparison shoppers. Thus, when manufacturers compete for the larger common demand, the marginal impact on demand of a decrease in price increases. Essentially, the informational product differentiation among non-new consumers (i.e., partially-informed and fully informed segments in the absence of the recommender system) is less with the recommender system than without. Two, the recommendation policy used by the retailer - recommend the product that offers a higher expected net consumer utility - introduces a subtle effect on the informational differentiation between the firms. Under this policy, a price decrease by a manufacturer will increase the recommendations for the manufacturer and therefore its informational advantage over the competitor. This provides an incentive for manufacturers to reduce their price to gain informational advantage among new consumers (i.e., the uninformed segment in the absence of the recommender system). This is seen from the firms' demand functions shown in Equation (12). Firm A's demand can be rewritten as the following: $\frac{1}{2}+\frac{(p_B-p_A)}{2t}+\frac{(1-\beta)(1-\theta-\theta_b)(p_B-p_A)}{\beta}$. The first two terms represent the demand function under full information. While the first term is the base demand, the second term is the substitution effect induced by price because of location differentiation (i.e., when there is no informational differentiation). The third term is the additional substitution effect induced by price because of informational differentiation. We note that common demand is $\theta+\theta_b$, and the manufacturers use price as a lever to attract more recommendations in their favor and increase their informational advantage among the rest of the consumer population. Proposition 1(b) shows that the substitution effect dominates the demand effect causing the price to be lower when the recommender system is present than when it is not.

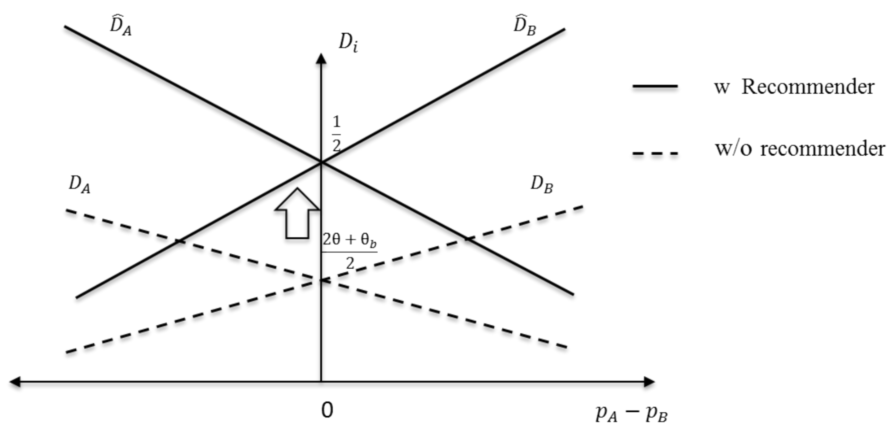


Figure 1: Demand effect and Substitution effect

Proposition 1(c) shows that the manufacturers and the retailer do not necessarily benefit from the recommender system. This is because while the demand effect benefits the sellers, the subtitution effect hurts them. The conditions in Proposition 1(c) for the retailer and manufacturers to benefit from the recommender system are low θ, low θ_b, and high β, which are illustrated in Figure 2. Intuitively, under these conditions, the demand effect of recommender system is likely to dominate the substitution effect. In particular, when the proportion of partially informed consumers $\theta > \frac{1}{8}$, the manufacturers are always worse off when the recommender system is used. When $\theta < \frac{1}{8}$, the proportion of fully informed consumers also has to be moderate (i.e., neither too large nor too low) for the recommender system to benefit the retailer and manufacturers. Furthermore, Figure

2 shows that the region where the retailer and manufacturers are better off in the presence of recommendation expands as β increases.

Proposition 1(d) and (e) show that the recommender system benefits consumers and the society. Social welfare benefits because, compared to the scenario without the recommender system, more consumers buy and realize a positive surplus, and more consumers buy the product that offers them a higher surplus when the recommender system is used. Additionally, consumers benefit from lower prices also.

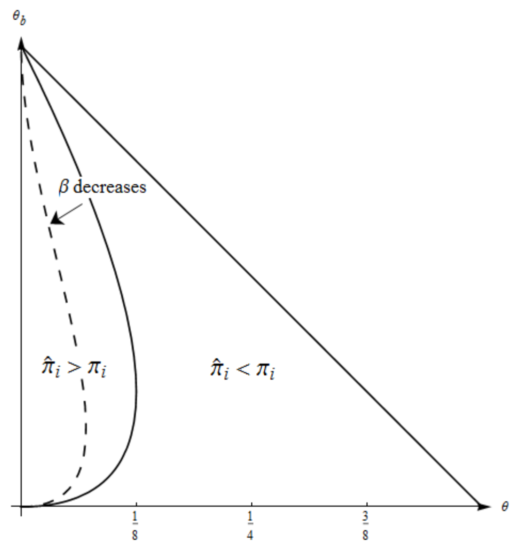


Figure 2: Impact of recommender system on profits

Next we analyze how the model parameters affect the impact of the recommender system on sellers. We denote the benefit from the recommender system to firm i as $\Delta\pi_i = \hat{\pi}_i - \pi_i$, $i \in \{A, B, R\}$. Because the recommender system does not always benefit the firms, $\Delta\pi_i$ can be positive or negative.

Proposition 2. *(a) An increase in β increases the benefit from the recommender system (i.e., $\frac{\partial \Delta\pi_i}{\partial \beta} \geq 0$).*

(b) An increase in θ decreases the benefit from the recommender system (i.e., $\frac{\partial \Delta\pi_i}{\partial \theta} \leq 0$) if and only if $4(1 + \frac{2\theta}{\theta_b}) \geq \frac{(1-\beta)\beta}{(1-(\theta+\theta_b)(1-\beta))^2}$.

(c) An increase in θ_b increases the benefit from the recommender system (i.e., $\frac{\partial \Delta\pi_i}{\partial \theta_b} \geq 0$), if and only if $1 - \frac{4\theta^2}{\theta_b^2} \leq \frac{(1-\beta)\beta}{(1-(\theta+\theta_b)(1-\beta))^2}$.

Proposition 2(a) shows that the retailer and manufacturers always benefit from improving the recommender system precision. Specifically, if the recommender system benefits the retailer, an improvement in β enhances the benefit; if the recommender system hurts the retailer, an improvement in β mitigates this negative impact. Figure 3 illustrates the intuition for the impact of an improvement in recommender system precision. We note that an improvement in recommender system precision does not change the demand effect—the total market size is unaffected by the precision when the recommender system is used. Consequently, the precision affects only the substitution effect of the recommender system. As seen from Figure 3, an improvement in the recommender system precision reduces the elasticity of the demand function, which softens the price competition. This surprising result is because of an intricate interaction between the recommender system precision and the substitution effect. Regardless of the recommender system precision, a decrease in the price of a product, ceteris paribus, increases the likelihood of that product being recommended. However, the marginal increase in this likelihood is higher when the recommender system precision is low compared to when the precision is high. When the precision is low, from the the recommender system's perspective, most consumers are concentrated in the middle of the Hotelling line, and therefore a small decrease in the price of a product will induce recommendation of that product to a large number of consumers. For instance, suppose the recommendation system precision is zero, then the recommender system will recommend the lower priced product to every consumer and this provides each manufacturer significant incentives to undercut the competition. On the other hand, when the precision is high, from the recommender system's perspective, the consumers are located more evenly throughout the line, and therefore, a decrease in price leads to recommendation of that product to a smaller number of consumers. In the extreme case of perfect precision, the recommender system has full information about consumers, and therefore only location differentiation contributes to substitution effect induced by price resulting in the same equilibrium as the full information model. Essentially, the recommender system assigns a larger weight to the location signal relative to prodct prices when the precision is high while choosing the recommendation. Consequently, manufacturers engage in more intense price cuts when precision is low compared to when precision is high.

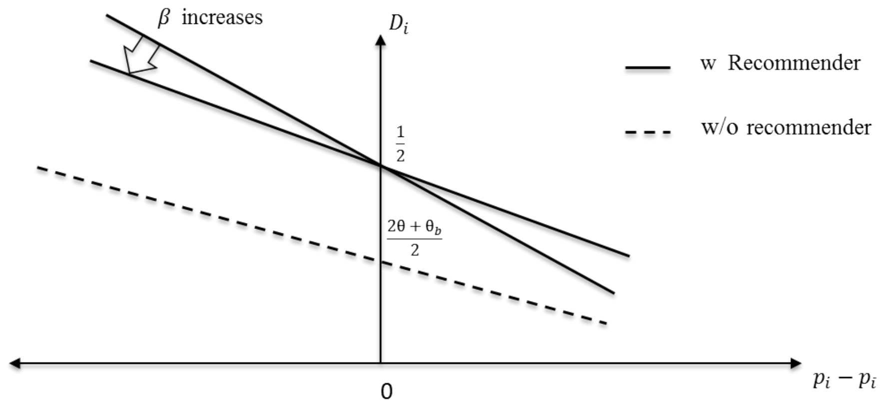


Figure 3: Impact of recommender system precision

Proposition 2(b) shows that the impact of θ can either enhance or diminish the benefit from recommender system. As illustrated in Figure 4, an increase in θ diminishes the demand effect; that is, the gap between the demand intercepts with and without the recommender system decreases as θ increases. Therefore, the positive effect of the recommender system decreases as θ increases. Meanwhile, an increase in θ diminishes the substitution effect by the change in the demand elasticity; that is, while the (absolute) slope of the demand function decreases as θ increases when the recommender system is present, the slope is unaffected by θ when the recommender system is absent. Therefore, the negative effect of the recommender system also decreases as θ increases. Whether an increase in θ enhances or reduces the benefit from the recommender system depends on the tradeoff between these two effects of θ. We find that if the recommender system is fairly precise (i.e., if $\beta \geq \frac{1}{5}$), an increase in θ always diminishes the benefit from recommender system.

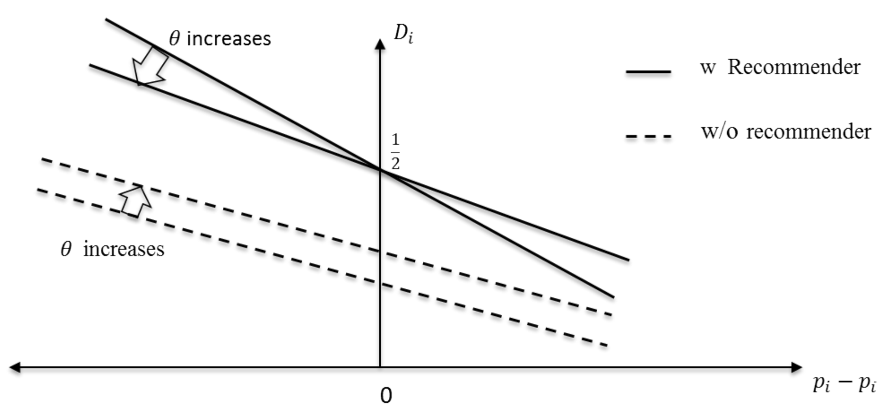


Figure 4: Impact of partially-informed consumers

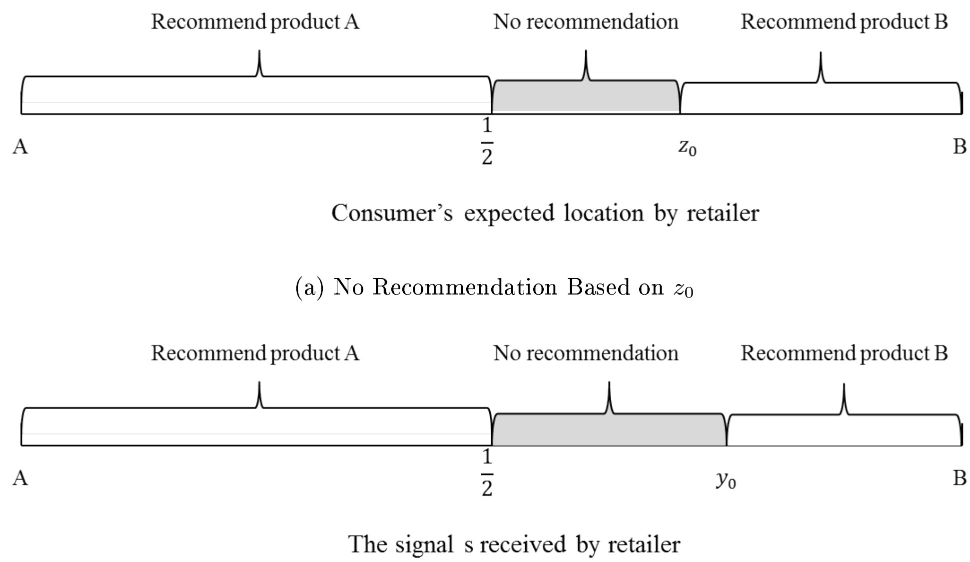
Similar to θ, an increase in θ_b diminishes the demand effect. An increase in θ_b may diminish the substitution effect also because an increase in θ_b increases the (absolute) slope of the demand function in the no-recommender but decreases the (absolute) slope in the recommender case. Therefore, an increase in θ_b may enhance or diminish the benefit from the recommender system depending on the tradeoff between the two.

In summary, the analysis in this section reveals that when manufacturers strategically respond to the deployment of recommender systems by the retailer, the retailer as well as manufacturers may not necessarily benefit even though the recommender system indeed increases the demand and total market size. While the retail platform can increase the benefit or decrease the loss from the deployment of the recommender system by increasing its precision, even a perfect precision cannot guarantee that it benefits from the recommender system. On the other hand, the recommender system always benefits consumers and the society implying that the retailer may indeed, under some conditions, benefit consumers at its own expense when it deploys the recommender system. Consequently, a natural question that arises is whether the retailer can enhance its own benefit or mitigate the loss while still following a recommendation strategy that has consumer interest at its core; that is, recommend only the product that offers a higher expected net utility for the consumer. We explore one such strategy next.

4 Partial Recommendation

The finding that the recommender system may hurt the retailer and manufacturers, on the one hand, raises questions regarding the long-run viability of such a recommender system when the sellers are hurt by it. On the other hand, it may induce the retailer to deploy a recommender system that maximizes its own payoff, but such a system may not be desirable in the long run because it may antagonize the consumers. In this section, we propose a variation of the recommendation strategy considered in the previous section, referred to as *partial recommendation*, that mitigates the negative effect of recommendations while retaining the positive aspect of providing recommendation based on consumer welfare.

The partial recommendation strategy does not guarantee that a recommendation is provided to every consumer. In particular, it does not recommend any product to consumers that are perceived to be located between the middle point of the consumer preference line (i.e., $\frac{1}{2}$) and the indifference point based on product prices (i.e., z_0). We refer to this part of the Hotelling line as the no-recommendation region. For consumers that are perceived to be located outside of this no-recommendation region, the recommender system recommends the product that offers a higher expected consumer net utility, as in the previous section. The recommendation and no-recommendation regions under the partial recommendation strategy are illustrated in Figure 5. In panel (a) of Figure 5 the no-recommendation region is depicted using consumers' true locations. When the retailer is uncertain about the consumer's true location, the retailer does not recommend a product if the expected location, based on the signal it receives about the consumer, falls in the range $[\frac{1}{2}, y_0]$ as depicted in panel (b) of Figure 5.



(a) No Recommendation Based on z_0

(b) No Recommendation Based on y_0

Figure 5: Partial Recommendation Strategy

The partial recommendation strategy is consistent with the retailer's goal of helping consumers find their best product, and the strategy of not recommending any product to the consumers that are believed to be in the no-recommendation region is consistent with this goal. Since consumers located in the no-recommendation region have weak preference (relative to those that are closer to the products) for one product over the other, even minor errors in estimating the consumer location can lead to a wrong recommendation to these consumers. By not recommending any product, the partial recommendation strategy may reduce the likelihood of wrong recommendation. Furthermore, for consumers believed to be located between $\frac{1}{2}$ and z_0 in Figure 5, if the retailer were to recommend a product to these consumers, it would recommend A because it offers a higher expected net utility. However, this recommendation conflicts with the consumer's preference for B based solely on misfit cost. The partial recommendation strategy ensures that the retailer recommends only to those consumers for whom there is no such conflict between the misfit cost and utility from the retailer's perspective. Finally, the partial recommendation strategy is consistent with practice in that recommender systems do not always provide recommendations to every one.

As in Section 3, we make technical assumptions to ensure "full market coverage" and ensure that competition plays a role in equilibrium prices. Specifically, we make the following assumptions.

Assumption 3: $\frac{2t\beta}{\theta+3\beta\theta-2\beta^2\theta+2\beta\theta_b}+t<v<\frac{2t\beta}{(\theta+3\beta\theta-2\beta^2\theta+2\beta\theta_b)\theta}+t$

Assumption 4: $\frac{3}{10}<\beta<1$ and $\frac{1}{5}<\theta_b<1$

The product demands can be derived in a manner similar to that discussed in Section 3. Without loss of generality, we assume $p_A \le p_B$, which implies that $z_0 \ge \frac{1}{2}$ and $y_0 \ge z_0$.

Consider the segment in which consumers are only aware of product A. A consumer whose location is less than z_0 would buy product A whether or not she gets a recommendation and regardless of the recommendation she gets. A consumer whose location is between z_0 and y_0 receives recommendation for product A and thus buys product A with probability $\frac{(1-\beta)}{2}$, and receives recommendation for product B and buys product B with probability $1-(1-\beta)y_0-\beta$. A consumer located between y_0 and 1 receives recommendation for product A and buys product A with probability $\frac{(1-\beta)}{2}$, and receives recommendation for product B and buys product B with probability $1-(1-\beta)y_0$. Combining various signal scenarios, we compute the demand functions for consumers who are only aware of product A as follows:

$$\begin{aligned} d_A &= \theta y_0+\theta z_0(1-y_0)(1-\beta) \\ d_B &= \theta(1-y_0)\left[1-z_0(1-\beta)\right] \end{aligned} \tag{19}$$

We can derive the demand functions for other segments analogously. By aggregating these demand functions, we can obtain the following demand functions for the two products. (We provide the detailed derivation in the Appendix.)

$$D_A=\begin{cases} \frac{1}{2}-\frac{(p_B-p_A)^2(1-\beta)\theta-(p_B-p_A)(\theta+3\beta\theta-2\beta^2\theta+2\beta\theta_b)t}{4t^2\beta} & p_A\le p_B \\ \\ \frac{1}{2}+\frac{(p_B-p_A)^2(1-\beta)\theta+(p_B-p_A)(2-3\theta+3\beta\theta-2\beta^2\theta+2\beta\theta_b-2\theta_b)t}{4t^2\beta^2} & p_B<p_A \end{cases} \tag{20}$$

$$D_B=\begin{cases} \frac{1}{2}+\frac{(p_A-p_B)^2(1-\beta)\theta+(p_A-p_B)(2-3\theta+3\beta\theta-2\beta^2\theta+2\beta\theta_b-2\theta_b)t}{4t^2\beta^2} & p_A\le p_B \\ \\ \frac{1}{2}-\frac{(p_A-p_B)^2(1-\beta)\theta-(p_A-p_B)(\theta+3\beta\theta-2\beta^2\theta-2\beta\theta_b)t}{4t^2\beta} & p_B<p_A \end{cases} \tag{21}$$

Manufacturers choose p_A and p_B simultaneously to maximize their profits, the pricing game is characterized by multiple symmetric equilibria. Specifically, any price in the range $\left[\frac{2t\beta}{2-\theta(3-3\beta+2\beta^2)-2\theta_b(1-\beta)}, \frac{2t\beta}{\theta(1+3\beta-2\beta^2)+2\beta\theta_b}\right]$ is an equilibrium for the game. We follow the frequently employed strategy of using the Pareto-optimal equilibrium as the focal equilibrium for our analysis. Lemma 3 summarizes the Pareto-optimal equilibrium.

Lemma 3. *When the retailer uses partial recommender system, the Pareto-optimal equilibrium prices, demands, manufacturer profits, retailer profit, consumer surplus, and social welfare are as follows:*

(a) *Price:*

$$p_A^* = p_B^* = \frac{2t\beta}{\theta(1+3\beta-2\beta^2)+2\beta\theta_b} \tag{22}$$

(b) *Demand:*

$$D_A^* = D_B^* = \frac{1}{2} \tag{23}$$

(c) *Manufacturer profit:*

$$\pi_A^* = \pi_B^* = \frac{t\beta(1-\alpha)}{\theta(1+3\beta-2\beta^2)+2\beta\theta_b} \tag{24}$$

(d) *Retailer profit:*

$$\pi_R^* = \frac{2t\beta\alpha}{\theta(1+3\beta-2\beta^2)+2\beta\theta_b} \tag{25}$$

(e) *Consumer surplus:*

$$CS = v - \frac{2t\beta}{\theta+3\beta\theta-2\beta^2\theta+2\beta\theta_b} - \frac{t}{4} - \frac{(1-\theta_b-\theta)(1-\beta)t}{4} \tag{26}$$

(f) *Social welfare:*

$$W = v - \frac{t}{4} - \frac{(1-\theta_b-\theta)(1-\beta)t}{4} \tag{27}$$

Proposition 3. *Compared to the full recommendation strategy, under partial recommendation,*

(a) *Each product's price is higher.*

(b) *Each product's demand is the same.*

(c) *The retailer and manufacturer's profits are higher.*

(d) *Customer surplus is lower.*

(e) *Social welfare is the same.*

Proposition 3(c) reveals that the retailer and manufacturers are better off under the partial recommendation strategy as compared to the full recommendation strategy. Clearly, the price competition is softened under the partial recommendation strategy compared to the full recommendation strategy (Proposition 3(a)), while the demand effect remains the same under both strategies (Proposition 3(b)). The reason for the less intense price competition under the partial recommendation strategy than under the full recommendation strategy can be explained using the different effects of the two strategies on the demand functions. Figure 6 illustrates how the strategies affect the demand functions. Clearly, the slope of the demand function is less steep near the equilibrium point ($p_A = p_B$) under the partial recommendation strategy than under the full recommendation strategy. This is because partial recommendation diminishes the incentive of manufacturers to reduce the price to generate more recommendation in their favor and create informational advantage—if y_0 is equal to or greater than $\frac{1}{2}$, then any price reduction by manufacturer A will not result in more recommendations in its favor, and similarly, if y_0 is equal to or less than $\frac{1}{2}$, then any price reduction by manufacturer B will not result in more recommendations in its favor. The softening of price competition by the partial recommendation strategy clearly hurts consumers (Proposition 3(d)) but does not affect the social welfare (Proposition 3(e)).

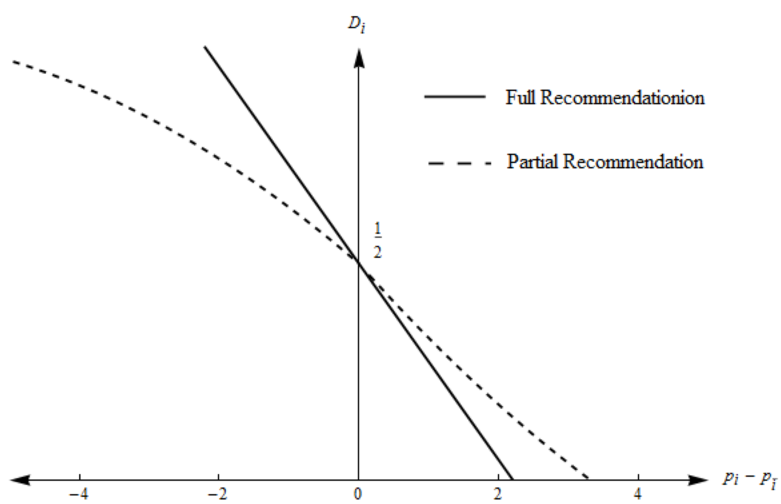


Figure 6: Demand under partial recommender system ($\beta = \frac{3}{10}$, $\theta = \frac{1}{4}$,$\theta_b = \frac{2}{5}$, $t = 4$)

The following result compares the partial recommendation strategy with no recommendation.

Proposition 4. *Compared to the scenario without the recommender system, in the scenario with the partial recommender system,* *(a) Each product's demand is higher.*

(b) Each product's price is lower if $\theta_b \leq (\theta+\theta_b)(2\theta+\theta_b)$ *regardless of the recommender system precision; Each product's price is higher if and only if* $\theta_b > (\theta+\theta_b)(2\theta+\theta_b)$ *and* $\beta > \frac{6\theta^2+7\theta\theta_b-2(1\text{-}\theta_b)\theta_b+\sqrt{8\theta^2(2\theta+\theta_b)^2+\left[6\theta^2+7\theta\theta_b-2(1-\theta_b)\theta_b\right]^2}}{4\theta(2\theta+\theta_b)}$

(c) The retailer and manufacturers are worse off if $\theta_b \leq (2\theta+\theta_b)^2(\theta+\theta_b)$ *regardless of the recommender system precision; The retailer and manufacturers are better off if and only if* $\theta_b \leq (2\theta+\theta_b)^2(\theta+\theta_b)$ *and* $\beta > \frac{(3\theta+2\theta_b)(2\theta+\theta_b)^2-2\theta_b-\sqrt{[(3\theta+2\theta_b)(2\theta+\theta_b)^2-2\theta_b]^2+8\theta^2(2\theta+\theta_b)^4}}{4\theta(2\theta+\theta_b)^2}$

(d)Consumer surplus is lower if and only if $v < \tilde{v}$ *and* $\tilde{v} > \max[\frac{2(\theta+\theta_b)t}{\theta_b}, \frac{2t\beta}{\theta+3\beta\theta-2\beta^2\theta+2\beta\theta_b}+t]$, *where* $\tilde{v} = \frac{t}{1-2\theta-\theta_b}\left[\frac{2\beta}{\theta+3\beta\theta-2\beta^2\theta+2\beta\theta_b}+\frac{1}{4}+\frac{(1-\theta_b-\theta)(1-\beta)}{4}-\frac{(2\theta+\theta_b)^2}{\theta_b}\right]$

(e) Social welfare is higher.

We find that the comparisons between the no-recommendation scenario and the partial recommendation scenario are qualitatively similar to those between the no-recommendation scenario and the full recommendation scenario with one important difference. Recall from Proposition 1(b) that full recommendation always intensifies the price competition between manufacturers. However, the price competition can be less intense under partial recommendation than under no recommendation if the size of the partially informed consumer segment is sufficiently low relative to the size of fully informed consumer segment and the recommender system precision is sufficiently high. The effect of partial recommendation on the slope of the demand function, discussed following Proposition 3, explains this result also.

The most significant implication of the analysis in this section is that by using the partial recommendation strategy, i.e., by committing to not recommend any product to consumers for whom the misfit cost and utility conflict with other from the recommender system's perspective, the retailer can increase the benefit or mitigate the loss from the recommender system.

5 Model Extension

The model analyzed in Sections 3 and 4 considers symmetric manufacturers. In the extension, we allow for asymmetry in consumer awareness about the two products. Without loss of generality, we assume that product A has a higher awareness level among consumers. Let the fraction of consumers who are only aware of product A be $\theta+u$, the fraction of consumers who are only aware of product B be $\theta-u$, and the fraction of consumers who are aware of both product be θ_b, where $0 \leq u \leq \min\{\theta, 1-\theta\}$. The parameter u measures the extent of informational advantage of A over

B. For ease of exposition, we present only the results for the case when the recommender system has perfect precision. Following a derivation procedure similar to the one discussed in Section 3, we show the following result.

Lemma 4. *When the consumer awareness about the two products is asymmetric, the equilibrium prices, demands, manufacturer profits, retailer profit, consumer surplus, and social welfare are as follows:*

	without recommendation	*with recommendation under perfect precision*
p_A^*	$\frac{t(6\theta+2u+3\theta_b)}{3\theta_b}$	t
p_B^*	$\frac{t(6\theta-2u+3\theta_b)}{3\theta_b}$	t
D_A^*	$\frac{1}{6}(6\theta+2u+3\theta_b)$	$\frac{1}{2}$
D_B^*	$\frac{1}{6}(6\theta-2u+3\theta_b)$	$\frac{1}{2}$
π_A^*	$\frac{(1-\alpha)t(6\theta+2u+3\theta_b)^2}{18\theta_b}$	$\frac{(1-\alpha)t}{2}$
π_B^*	$\frac{(1-\alpha)t(6\theta-2u+3\theta_b)^2}{18\theta_b}$	$\frac{(1-\alpha)t}{2}$
π_R^*	$\alpha t\frac{4u^2+9(2\theta+\theta_b)^2}{9\theta_b}$	αt
CS	$v(2\theta+\theta_b)-t(5\theta+\frac{5\theta_b}{4}+\frac{4(2u^2+9\theta^2)}{9\theta_b})$	$v-\frac{5t}{4}$
W	$v(2\theta+\theta_b)-t(\theta+\frac{\theta_b}{4}+\frac{4u^2}{9\theta_b})$	$v-\frac{t}{4}$

When there is no recommender system, manufacturer A has a higher price, higher demand, and higher profit than manufacturer B, and the differences in prices, demands, and profits are increasing in the informational advantage. The retailer's profit is also increasing in the informational advantage. This is because an increase in the informational advantage one manufacturer enjoys over the other reduces the price competition between the two. Furthermore, more consumers buy the higher-priced product which also benefits the retailer. On the other hand, a higher informational advantage for one product hurts the consumers and social welfare. When the retailer deploys a recommender system with perfect precision, every consumer buys the recommended product. Since all consumers are recommended a product, the awareness levels are equalized and therefore the informational advantage enjoyed by manufacturer A in the absence of a recommender system does not play any role in the equilibrium quantities.

Proposition 5. *Under asymmetric awareness, compared to the scenario without the recommender system, in the presence of the full recommender system:*

(a) *Demand for product A is higher if and only if* $u \leq \frac{3}{2}(1-(2\theta+\theta_b))$ *and demand for product B is always higher.*

(b) *Each product's price is lower.*

(c) *Manufacturer A's profit is higher if and only if* $\theta \leq \frac{\sqrt{\theta_b}}{2}(1-\sqrt{\theta_b})$, *and* $u \leq \frac{3(\sqrt{\theta_b}-(2\theta+\theta_b))}{2}$,

(d) *Manufacturer B's profit is lower if and only if* $\theta \geq \frac{\sqrt{\theta_b}}{2}(1-\sqrt{\theta_b})$ *and* $u \leq \frac{3((2\theta+\theta_b)-\sqrt{\theta_b})}{2}$,,

(e) *Retailer R's profit is higher if and only if* $\theta \leq \frac{\sqrt{\theta_b}}{2}(1-\sqrt{\theta_b})$ *and* $u \leq \frac{3\sqrt{\theta_b-(2\theta+\theta_b)^2}}{2}$.

(f) *Consumer surplus is higher.*

(g) *Social welfare is higher.*

We find that the impacts of the recommender system when the consumer awareness levels about the two products are asymmetric are generally qualitatively similar to those when the consumer awareness levels are symmetric. In particular, the impacts of recommender system on price, retailer's profit, consumer surplus, and social welfare are qualitatively identical in both symmetric awareness and asymmetric awareness models. The key additional insights we derive relate to how the extent of informational advantage enjoyed by one of the manufacturers (i.e., u) affects the results. We find that u does not change the impact of recommender system on the price competition, consumer surplus, and social welfare qualitatively. While the recommender system always increases the demand of the product that has the lower consumer awareness, it decreases the demand of the product that has the higher consumer awareness if u is high. On the profit side, the retailer and the manufacturer that has the higher awareness are hurt when the difference in awareness levels is high, but the manufacturer with the lower awareness level benefits in this case. The intuition for the differential impact of recommender system on the manufacturers when the difference in the awareness levels is high relates to the awareness (or information advantage) smoothing effect of recommender system. We note that the recommender system makes its recommendation based on expected consumer surplus which depends solely on consumer location and product prices, and therefore, the initial awareness level of consumer plays no role in recommendations. Consequently, the manufacturer with the higher awareness loses some of its informational advantage when the recommender is used; on the other hand, the disadvantage suffered by the manufacturer with the lower awareness is mitigated by the recommender system. From the retailer's perspective, a higher level of information advantage helps. A higher level of advantage reduces the price competition because the manufacturer that has a the advantage can charge a higher price. Moreover, the manufacturer with the advantage has a higher demand. Therefore, more consumers buying higher priced product helps the retailer when the informational advantage is high. This asymmetry is reduced by recommender system, which explains Proposition 5(d).

Proposition 5suggests that a manufacturer with the higher initial informational advantagewill prefer the recommender system only when both the average awareness level and the informational advantage are not high, a manufacturer with the informational disadvantage will prefer the recommender system only if either the average awareness level is low or the disadvantage is high, and that the retailer will prefer the recommender system only when both the average awareness level and the difference in awareness levels are not high.

6 Conclusions

We examine the effect of recommender system in a channel structure with a retail platform and two competing manufacturers selling substitute products. Manufacturers set the prices for their products and the retail platform takes a fraction of the sales price as commission fee for each transaction. The recommendations have an informative role and increase the consumer awareness of products. The retail platform recommends the product that offers the highest expected net utility to the consumer. The most important finding of our analysis is that the retail platform and the manufacturers do not always benefit from the recommender system. While the demand always increases, the price decreases when the recommender system is deployed. While strategies such as improving recommendation precision or committing to not always recommend to every consumer mitigate the loss from a recommender system, they may not completely offset the loss. The findings arise as result of the complex interplay of demand expansion enabled by recommender system, the substitution between firms because of location differentiation and informational differentiation, and the impact of price on the informational differentiation,

The findings have several implications for electronic marketplaces that deploy recommender systems. First, focusing solely on the additional sales or demand created by recommender systems and ignoring the possible strategic price responses from manufacturers that sell in the marketplace can hurt the retail platform that implements the recommender system. Furthermore, when the cost of developing recommender systems is also accounted for, the value of these system diminishes further. Second, since recommender systems have an advertising role, the manufacturers enjoy "free" advertising provided by the retail platform when it deploys recommender systems. A retail platform may have to reconsider its contract with the manufacturers in light of this advertising service it provides them and to benefit from recommender systems. Finally, the consumers are the beneficiaries of recommendations. It may be worthwhile for a retail platform to devise ways by which it can extract some of the surplus enjoyed by consumers because of recommender systems.

This study can be extended in several ways. The contexts and the manner in which recommender systems are employed vary widely. For instance, recommender systems vary in terms of the strategy and algorithms they use, when recommendations are presented to the user, how many recommendations are presented, and the role they play (whether informative or persuasive). They are deployed in both platform settings and resale settings. As a first study to examine the upstream impact of recommender systems, we have analyzed a specific context in this paper using a stylized model of two firms. Future research can extend the study by changing any of the above mentioned dimensions to enrich our understanding of the economic impact of recommendation systems.

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Appendix

A.1 Proof of Lemma 1

Proof. (a) Each manufacturer's best response to its competitor in stage 1 is characterized by the first-order conditions of Equation (3):

$$\frac{\partial \pi_A}{\partial p_A} = (1-\alpha)\left(\theta + \frac{p_B - 2p_A + t}{2t}\theta_b\right) = 0$$
$$\frac{\partial \pi_B}{\partial p_B} = (1-\alpha)\left(\theta + \frac{p_A - 2p_B + t}{2t}\theta_b\right) = 0$$

Based on these equations, we can derive the manufacturer equilibrium pricesas in Equation (4).

(b) Substituting the equilibrium prices into Equation (2), we can derive the equilibrium demands as in Equation (5).

(c) Substituting the equilibrium prices into Equation (3), we can derive the equilibrium profits as in Equation (6). One possible deviation is that one manufacturer increases its price such that it only serves consumers who are aware of its product. In this case, we can show that it is the best for the manufacturer to set the price to $(v-t)$ and its deviation profit is thus $(1-\alpha)(v-t)\theta$. Under the assumption $v < \frac{(2\theta+\theta_b)^2 t}{2\theta\theta_b} + t$, the deviation profit is less than its equilibrium profit. We can verify other possible deviations always lead to less profits.

(d) According the commission scheme, we have $\frac{\pi_R}{\pi_A+\pi_B} = \frac{\alpha}{1-\alpha}$. Based on π_i^*, we can derive π_R^* as in Equation (7).

(e) Notice that the assumption $v \geq \frac{2(\theta+\theta_b)t}{\theta_b}$ can guarantee that $v-t-p^* \geq 0$ such that even the consumer with the largest misfit cost has incentive to purchase if she is aware of the product. The consumer surplus from each partially-informed segment is:

$$CS_p = \theta \int_0^1 (v - p^* - zt)dz = \theta(v - p^* - \tfrac{t}{2}) \tag{28}$$

and from the fully-informed segment is

$$CS_f = \theta_b \left[\int_0^{\frac{1}{2}} (v - p^* - zt)dz + \int_{\frac{1}{2}}^1 [v - p^* - (1-z)t]\, dz \right] = \theta_b \left(v - p^* - \tfrac{t}{4}\right) \tag{29}$$

The consumer surplus from the uninformed segment is zero. Aggregating the consumer surplus from the two partially-informed segments and one fully-informed segment, we can derive the total surplus as in Equation (8).

(f) Social welfare is the sum of consumer surplus, manufacturers' profits, and retailer's profit; that is, $W = CS + \pi_A^* + \pi_B^* + \pi_R^*$, which can be derived as in Equation (9). □

A.2 Proof of conditional expectation of misfit

Proof. The cumulative density function of s, conditional on the consumer's true location λ being z, can be formulated as:

$$P(s \leqslant y | \lambda = z) = (1-\beta)y + \beta H(y - z) \tag{30}$$

where $H()$ is the Heaviside step function that evaluates to zero if the argument is negative, and to one otherwise. The corresponding probability density function is:

$$P(s = y | \lambda = z) = (1-\beta) + \beta\delta(y - z) \tag{31}$$

where $\delta(x)$ is the Dirac delta distribution that satisfies $\int_{-\infty}^{\infty} \delta(x)dx = 1$ and $\delta(x) = 0$ if $x \neq 0$,

$\delta(x) = \infty$ if $x = 0$.

Using the Bayes Law,

$$P(\lambda = z|s = y) = \frac{P(s = y|\lambda = z)P(\lambda = z)}{P(s = y)} = (1 - \beta) + \beta\delta(y - z) \tag{32}$$

and the conditional expectation is:

$$E(\lambda = z|s = y) = \frac{1 - \beta}{2} + \beta y$$

□

A.3 Proof of Lemma 2

Proof. The manufacturers maximize their profits by choosing their optimal prices; that is,

$$\max_{p_i} \pi_i = (1 - \alpha)p_i D_i \tag{33}$$

where D_i is defined in Equation (12).

(a) Each manufacturer's best response to its competitor in stage 1 is characterized by the first-order conditions of Equation (33):

$$\begin{aligned} \tfrac{\partial \pi_A}{\partial p_A} &= (1 - \alpha)\left[\tfrac{1}{2} + \tfrac{1-(1-\beta)(\theta+\theta_b)}{2t\beta}\Big(p_B - 2p_A\Big)\right] = 0 \\ \tfrac{\partial \pi_B}{\partial p_B} &= (1 - \alpha)\left[\tfrac{1}{2} + \tfrac{1-(1-\beta)(\theta+\theta_b)}{2t\beta}\Big(p_A - 2p_B\Big)\right] = 0 \end{aligned}$$

Based on these equations, we can derive the manufacturer equilibrium prices as in Equation (13).

(b) Substituting the equilibrium prices into Equation (12), we can derive the equilibrium demands as in Equation (14).

(c) Substituting the equilibrium prices into Equation (33), we can derive the equilibrium profits as in Equation (15). Next we use firm A to show there is no profitable deviation from the equilibrium. One possible deviation for A is to reduce the price to $p \in (p_B^* - t, p_B^* - \beta t)$ such that $y_0 > 1$ and the recommender system always recommends B. In this case, the demand for A is $D_A = (\theta + \theta_b)\frac{p_B^* - p_A + t}{2t}$ and $\pi_A = (1 - \alpha)p_A D_A$. The optimal deviation price is $\frac{p_B^* + t}{2}$ and the maximum deviation profit is $\pi_A^{dev} = (1 - \alpha)\frac{\theta+\theta_b}{8t}\left(p_B^* + t\right)^2$. $\pi_A^{dev} > \pi_A^*$ requires

$$\beta > \tfrac{2\sqrt{1-\theta-\theta_b} - (1-\theta-\theta_b)(2+\theta+\theta_b)}{(\theta+\theta_b)(3+\theta+\theta_b)} \equiv \tilde{\beta}$$

We can calculate that $\max_{\theta,\theta_b}(\tilde{\beta}) = \frac{1}{9}$. Therefore, when $\beta > \frac{1}{9}$, manufacturer A has no incentive to deviate. We can verify other possible deviations always lead to less profits.

(d) According the commission scheme, we have $\frac{\pi_R}{\pi_A+\pi_B} = \frac{\alpha}{1-\alpha}$. Based on π_i^*, we can derive π_R^* as in Equation (16).

(e) As we shall show in the proof of Proposition 1 equilibrium price in the presence of the recommender system is lower than that without the recommender system. Because all the informed consumers are served without the recommender system, all the consumers are also served in the presence of the recommender system. The consumer surplus from each partially-informed segment is:

$$\begin{aligned} CS_p &= \theta\left[\int_0^{\frac{1}{2}}(v-p^*-zt)dz + \tfrac{1-\beta}{2}\int_{\frac{1}{2}}^1(v-zt-p^*)dz + \tfrac{1+\beta}{2}\int_{\frac{1}{2}}^1[v-(1-z)t-p^*]dz\right] \\ &= \theta\left[v-p^*-\tfrac{t}{2}+\tfrac{(1+\beta)t}{8}\right] \end{aligned} \tag{34}$$

The consumer surplus from the fully-informed segment is

$$CS_f = \theta_b\left[\int_0^{\frac{1}{2}}(v-p^*-zt)dz + \int_{\frac{1}{2}}^1\left[v-p^*-(1-z)t\right]dz\right] = \theta_b(v-p^*-\tfrac{t}{4}) \tag{35}$$

The consumer surplus from the uninformed segment is

$$\begin{aligned} CS_u &= 2(1-2\theta-\theta_b)\left[\tfrac{1+\beta}{2}\int_0^{\frac{1}{2}}(v-p^*-zt)dz + \tfrac{1-\beta}{2}\int_0^{\frac{1}{2}}[v-p^*-(1-z)t]dz\right] \\ &= (1-2\theta-\theta_b)\left[v-p^*-\tfrac{t}{4}-\tfrac{(1-\beta)t}{4}\right] \end{aligned} \tag{36}$$

Aggregating the consumer surplus from the four segments, we can derive the total surplus as in Equation (17).

(f) Social welfare is the sum of consumer surplus, manufacturers' profits, and retailer's profit; that is, $W = CS + \pi_A^* + \pi_B^* + \pi_R^*$, which can be derived as in Equation (18). □

A.4 Proof of Proposition 1

Proof. (a) Notice that

$$\frac{\partial \hat{p}_i}{\partial \beta} = \frac{t(1-\theta-\theta_b)}{[1-(1-\beta)(\theta+\theta_b)]^2} > 0 \tag{37}$$

and thus $\hat{p}_i$ increases in β and achieves its maximum at $\beta = 1$. Notice $\hat{p}_i|_{\beta=1} = t$, which is less than p_i. Therefore, $\hat{p}_i < p_i$.

(b) Because $2\theta + \theta_b \le 1$, $d_i = \frac{2\theta+\theta_b}{2} \le \frac{1}{2} = \hat{d}_i$.

(c) Because $\hat{\pi}_i = (1-\alpha)\hat{p}_i/2$ and $\frac{\partial \hat{p}_i}{\partial \beta} > 0$, we have $\frac{\partial \hat{\pi}_i}{\partial \beta} > 0$. Therefore, if $\hat{\pi}_i|_{\beta=1} < \pi_i$, equivalently, if $\theta > \frac{1}{2}\left(\sqrt{\theta_b} - \theta_b\right)$, $\hat{\pi}_i < \pi_i$. If $\theta < \frac{1}{2}\left(\sqrt{\theta_b} - \theta_b\right)$,

$$\hat{\pi}_i = \frac{(1-\alpha)t\beta}{2[1-(1-\beta)(\theta+\theta_b)]} \geq \pi_i = \frac{(1-\alpha)(2\theta+\theta_b)^2 t}{2\theta_b}$$

requires $\beta \geq \frac{(2\theta+\theta_b)^2(1-\theta-\theta_b)}{\theta_b-(2\theta+\theta_b)^2(\theta+\theta_b)}$. Combining the condition required in Lemma 2, $\hat{\pi}_i < \pi_i$ requires the condition specified in Part (c).

(d) Because $\hat{p}_i < p_i$, based on Equations (28) and (34) and Equations (29) and (35), we have $CS_p < \hat{CS}_p$ and $CS_f < \hat{CS}_f$. In addition, $CS_u = 0 < \hat{CS}_u$. Therefore, $\hat{CS} \geq CS$.

(e) Notice that social welfare generated by a consumer from consuming a product is the total value $v - zt$, which differs from the consumer's surplus $v - p^* - zt$ by p^*. Based on Equations (28) and (34), we have $W_p = \theta(v - \frac{t}{2})$ and $\hat{W}_p = \theta(v - \frac{t}{2} + \frac{(1+\beta)t}{8})$. Therefore, $W_p < \hat{W}_p$. Based on Equations (29) and (35), we have $W_f = \hat{W}_f$. In addition, $W_u = 0 < \hat{W}_u$. Therefore, $\hat{W} \geq W$. □

A.5 Proof of Proposition 2

Proof. (a) Notice that $\hat{\pi}_i = (1-\alpha)\hat{p}_i/2$. Therefore,

$$\frac{\partial \triangle \pi_i}{\partial \beta} = \frac{\partial \hat{\pi}_i}{\partial \beta} = \frac{(1-\alpha)}{2}\frac{\partial \hat{p}_i}{\partial \beta} > 0 \tag{38}$$

where the last inequality is because of Equation (37).

(b) Notice that

$$\frac{\partial \triangle \pi_i}{\partial \theta} = \frac{(1-\alpha)t}{2}\left[\frac{(1-\beta)\beta}{[1-(1-\beta)(\theta+\theta_b)]^2} - \frac{4(2\theta+\theta_b)}{\theta_b}\right] \tag{39}$$

Therefore, if and only if $4(1+\frac{2\theta}{\theta_b}) > \frac{(1-\beta)\beta}{[1-(1-\beta)(\theta+\theta_b)]^2}$, $\frac{\partial \triangle \pi_i}{\partial \theta} < 0$.

(c) Notice that

$$\frac{\partial \triangle \pi_i}{\partial \theta_b} = \frac{(1-\alpha)t}{2}\left[\frac{(1-\beta)\beta}{[1-(1-\beta)(\theta+\theta_b)]^2} - \left(1 - \frac{4\theta^2}{\theta_b^2}\right)\right] \tag{40}$$

Therefore, if and only if $(1 - \frac{4\theta^2}{\theta_b^2}) \leq \frac{(1-\beta)\beta}{[1-(1-\beta)(\theta+\theta_b)]^2}$, $\frac{\partial \triangle \pi_i}{\partial \theta_b} \geq 0$. □

A.6 Demand Function under Partial Recommendation

First we drive the demand functions when $p_A \leq p_B$. For the consumers who are only aware of product A, the demand functions are derived as in Equation19 For the fully-informed consumers, partial recommender system does not affect the functions in the baseline case (i.e., $d_A = \theta_b z_0$ and $d_B = \theta_b(1 - z_0)$).

For consumers who are only aware of product B, a consumer located in interval $\left[0, \frac{1}{2}\right]$ receives recommendation for product A and buys A with probability $P(s \leq \frac{1}{2} | \lambda = z, 0 < z \leq \frac{1}{2}) = (1 + \beta)\frac{1}{2}$; Otherwise, the consumer buys product B. A consumer located in interval $\left[\frac{1}{2}, z_0\right]$ receives product A and buys product A with probability $P(s \leq \frac{1}{2} | \lambda = z, \frac{1}{2} < z \leq z_0) = (1 - \beta)\frac{1}{2}$; Otherwise, the consumer buys product B. A consumer located between z_0 and 1 buys product B regardless of the recommendation. All together, we have the demand functions from this segment as:

$$\begin{aligned} d_A &= \tfrac{\theta[(1-\beta)z_0+\beta]}{2} \\ d_B &= \theta - \tfrac{\theta[(1-\beta)z_0+\beta]}{2} \end{aligned}$$

A uninformed consumer purchases the product that is recommended to her. Noticing that half of them are recommended product A and $(1 - y_0)$ are recommended product B, we have the demand function as

$$\begin{aligned} d_A &= \tfrac{1-2\theta-\theta_b}{2} \\ d_B &= (1 - 2\theta - \theta_b)(1 - y_0) \end{aligned}$$

The demand functions are:

The total demand function is derived by aggregating the demand under each consumer segment. Similarly, we can derive the demand functions when $p_A > p_B$. All together, we have the demand functions as follows.

$$D_A = \begin{cases} \frac{2t^2\beta-(p_B-p_A)^2(1-\beta)\theta+(p_B-p_A)t(\theta+3\beta\theta-2\beta^2\theta+2\beta\theta_b)}{4t^2\beta} & \text{if } p_A \leq p_B \\ \frac{2t^2\beta+(p_B-p_A)^2(1-\beta)\theta+(p_B-p_A)t(2-3\theta+3\beta\theta-2\beta^2\theta+2\beta\theta_b-2\theta_b)}{4t^2\beta^2} & \text{if } p_B < p_A \end{cases} \tag{41}$$

$$D_B = \begin{cases} \frac{2t^2\beta+(p_A-p_B)^2(1-\beta)\theta+(p_A-p_B)t(2-3\theta+3\beta\theta-2\beta^2\theta+2\beta\theta_b-2\theta_b)}{4t^2\beta^2} & \text{if } p_A \leq p_B \\ \frac{2t^2\beta-(p_A-p_B)^2(1-\beta)\theta+(p_A-p_B)t(\theta+3\beta\theta-2\beta^2\theta-2\beta\theta_b)}{4t^2\beta} & \text{if } p_B < p_A \end{cases} \tag{42}$$

A.7 Proof of Lemma 3

Proof. We first consider the case with $p_B - \beta t \leq p_A < p_B \leq p_A + \beta t$ such that $0 \leq z_0 \leq 1$ and $0 \leq y_0 \leq 1$. Manufacturer A's best response to its competitor in stage 1 is characterized by the first-order conditions of Equation (3) (with D_A specified in Equation (41)):

$$\frac{\partial \pi_A}{\partial p_A} = \frac{2t^2\beta - \left(3p_A^2 - 4p_A p_B + p_B^2\right)(1-\beta)\theta + t(p_B - 2p_A)(\theta + 3\beta\theta - 2\beta^2\theta + 2\beta\theta_b)}{4\beta t^2} = 0 \tag{43}$$

We consider symmetric equilibrium with $p_A = p_B$. With the first-order condition above, we can derive $p_A^* = p_B^* = p^* = \frac{2t\beta}{\theta(1+3\beta-2\beta^2)+2\beta\theta_b}$. To make sure the market is fully covered, we need $v - p_i^* - t > 0$, which requires $v > \frac{2t\beta}{\theta+3\beta\theta-2\beta^2\theta+2\beta\theta_b} + t$. Next we consider the possible deviations and derive the conditions for non-deviation.

(a) Manufacturer A increases price from p_A^* to $p \in [p_B^* + \beta t, p_B^* + t]$ such that $y_0 < 0$ and $z_0 \geq 0$. In this case, recommender system never recommends A and we can derive the demand function for A as:

$$D_A = \theta \left[z_0 + \left(\tfrac{1}{2} - z_0\right) \tfrac{1+\beta}{2} + \tfrac{1}{2}\tfrac{1-\beta}{2} \right] + \theta_b z_0$$

By the first-order condition, we can derive the best response function of A as:

$$\hat{p}_A = \frac{p_B + t}{2} + \frac{t\theta}{\theta(1-\beta) + 2\theta_b} \tag{44}$$

By substituting p_B^*, we can derive the optimal deviation price and optimal deviation profit:

$$\hat{p}_A = t\left(\tfrac{\beta}{\theta+3\beta\theta-2\beta^2\theta+2\beta\theta_b} + \tfrac{\theta}{(1-\beta)\theta+2\theta_b} + \tfrac{1}{2}\right)$$
$$\hat{\pi}_A = t\frac{\left(4\beta\theta_b^2+2\theta_b(2\beta-2\beta^2+6\beta\theta+\theta)+3\theta^2+\theta\beta(\beta\theta(2\beta-9)-2\beta+8\theta+2)\right)^2}{16((1-\beta)\theta+2\theta_b)(\theta+3\beta\theta-2\beta^2\theta+2\beta\theta_b)^2}$$

$\hat{\pi}_A < \pi_A^*$ requires the following conditions:

$$\frac{4\theta^2}{(1-\beta)\theta+2\theta_b} + \frac{4\beta((\beta-2)\beta-1)\theta}{(\theta+3\beta\theta-2\beta^2\theta+2\beta\theta_b)^2} + \frac{4(\beta(3-\beta\theta)+\theta)}{(\beta(2\beta-3)-1)\theta-2\beta\theta_b} - \beta\theta + 5\theta + 2\theta_b + 4 < 0 \tag{45}$$

We notice that the left-hand side of Inequality (45) is increasing in θ, which reaches its maximum at $\theta = \frac{1-\theta_b}{2}$ and the maximum is

$$\tfrac{2(\theta_b-1)^2}{\beta(\theta_b-1)+3\theta_b+1} + \tfrac{1}{2}\beta(\theta_b-1) - \tfrac{8\beta((\beta-2)\beta-1)(\theta_b-1)}{(\beta(2\beta(\theta_b-1)+\theta_b+3)-\theta_b+1)^2} + \tfrac{4(\beta(\beta(-\theta_b)+\beta-6)+\theta_b-1)}{\beta(2\beta(\theta_b-1)+\theta_b+3)-\theta_b+1} - \tfrac{5(\theta_b-1)}{2} + 2\theta_b + 4$$

When $\frac{1}{5} < \theta_b < 1$ and $\frac{3}{10} < \beta < 1$, we can verify the above expression is always negative, which means the deviation is not profitable..

(b) Manufacturer A increases its price to $p_A > p_B^* + t$ such that $y_0 < 0$ and $z_0 < 0$. In this case, $d_A = \frac{\theta}{2}$ and $\pi_A = p_A \frac{\theta}{2}$. Then A choose the highest price ($\hat{p}_A = v - t$) and $\hat{\pi} = (v-t)\frac{\theta}{2}$. Then if $\hat{\pi} = (v-t)\frac{\theta}{2} < \pi_A^* = \frac{p_A^*}{2}$requires $v < \frac{2t\beta}{(\theta+3\beta\theta-2\beta^2\theta+2\beta\theta_b)\theta} + t$. We can verify other possible deviations always lead to less profits.

□

A.8 Proof of Proposition 3

Proof. (a) Notice that

$$\hat{p}_i - p_i = \frac{2t\beta}{\theta + 3\beta\theta - 2\beta^2\theta + 2\beta\theta_b} - \frac{t\beta}{1-(1-\beta)(\theta+\theta_b)} \tag{46}$$

$$= \frac{\beta t\left[(2\beta^2 - \beta - 3)\theta - 2\theta_b + 2\right]}{(\theta + 3\beta\theta - 2\beta^2\theta + 2\beta\theta_b)\left[1-(1-\beta)(\theta+\theta_b)\right]} \tag{47}$$

Because $(2\beta^2 - \beta - 3)\theta - 2\theta_b + 2 \geq -4\theta - 2\theta_b + 2 = 2\left[1-(2\theta+\theta_b)\right] \geq 0$, $\hat{p}_i - p_i \geq 0$.

(b) Part (b) is because $\hat{D}_i = D_i = \frac{1}{2}$.(c) Because $\pi_i = p_i D_i$ for $i \in \{A, B\}$, the comparison of $\hat{\pi}_i$ and π_i is the same as the price comparison. The same applies to the comparison of retailer's profits.

(d) Notice the consumer surplus takes the same form as in Equations (34), (35), and (36) by replacing p_i with $\hat{p}_i$. Because $p_i < \hat{p}_i$, $CS > \hat{CS}$.

(e) Notice that in equilibrium under partial recommendation each consumer buy the product from the same manufacturer as in the baseline case. Because social welfare is the total value created by from the consumption, the social welfare remains the same. □

A.9 Proof of Proposition 4

Proof. (a) Part (a) is because $\hat{D}_i - D_i = \frac{1}{2} - (\theta + \frac{\theta_b}{2}) \leq 0$

(b)Note that

$$\hat{p}_i - p_i = \frac{2t\beta}{\theta + 3\beta\theta - 2\beta^2\theta + 2\beta\theta_b} - \frac{(2\theta+\theta_b)t}{\theta_b} \tag{48}$$

$$= \frac{t\left[2\beta^2\theta(2\theta+\theta_b) + \beta\left[2\theta_b - (2\theta+\theta_b)(3\theta+2\theta_b)\right] - \theta(2\theta+\theta_b)\right]}{\theta_b\left(\theta + 3\beta\theta - 2\beta^2\theta + 2\beta\theta_b\right)} \tag{49}$$

The difference of prices increases in β and achieves its maximum at $\beta = 1$. $\hat{p}_i - p_i|_{\beta=1} = \frac{t}{\theta+\theta_b} - \frac{(2\theta+\theta_b)t}{\theta_b} = \frac{t\left[\theta_b - (\theta+\theta_b)(2\theta+\theta_b)\right]}{\theta_b(\theta+\theta_b)}$

When $\theta_b - (\theta+\theta_b)(2\theta+\theta_b) < 0$, $\hat{p}_i - p_i < 0$. $\hat{p}_i$ is always less than p_i.

When $\theta_b - (\theta+\theta_b)(2\theta+\theta_b) > 0$, $\hat{p}_i$ can either be greater or less than $p_i$depending on the value of β.Solve $\hat{p}_i - p_i = 0$. We can get two roots:

$$\beta_p^1 = \frac{6\theta^2 + 7\theta\theta_b - 2(1\text{-}\theta_b)\theta_b - \sqrt{8\theta^2(2\theta+\theta_b)^2 + \left[6\theta^2 + 7\theta\theta_b - 2(1-\theta_b)\theta_b\right]^2}}{4\theta(2\theta+\theta_b)}$$

$$\beta_p^2 = \frac{6\theta^2 + 7\theta\theta_b - 2(1\text{-}\theta_b)\theta_b + \sqrt{8\theta^2(2\theta+\theta_b)^2 + \left[6\theta^2 + 7\theta\theta_b - 2(1-\theta_b)\theta_b\right]^2}}{4\theta(2\theta+\theta_b)}$$

β_p^1 is always less than zero. When$\beta > \beta_p^2$, $\hat{p}_i$ is greater than p_i.

(c)

$$\hat{\pi}_i - \pi_i = \frac{t\beta}{\theta + 3\beta\theta - 2\beta^2\theta + 2\beta\theta_b} - \frac{(2\theta+\theta_b)^2 t}{2\theta_b} \quad (50)$$

$$= \frac{t\left[2\beta^2\theta(2\theta+\theta_b)^2 + \beta[2\theta_b - (2\theta+\theta_b)^2(3\theta+2\theta_b)] - \theta(2\theta+\theta_b)^2\right]}{2\theta_b\,(\theta + 3\beta\theta - 2\beta^2\theta + 2\beta\theta_b)} \quad (51)$$

The difference of profits increases in β and achieves its maximum at $\beta = 1$. $\hat{\pi}_i - \pi_i|_{\beta=1} = \frac{t}{2(\theta+\theta_b)} - \frac{(2\theta+\theta_b)^2 t}{2\theta_b} = \frac{t\left[\theta_b - (\theta+\theta_b)(2\theta+\theta_b)^2\right]}{2\theta_b(\theta+\theta_b)}$.

When $\theta_b - (\theta+\theta_b)(2\theta+\theta_b)^2 < 0$, $\hat{\pi}_i - \pi_i < 0$, that is, $\hat{\pi}_i$ is always less than π_i.

When $\theta_b - (\theta+\theta_b)(2\theta+\theta_b)^2 > 0$, $\hat{\pi}_i$ can either be greater than or less than π_i depending on β. Solve $\hat{\pi}_i - \pi_i = 0$. We get two roots:

$$\beta_\pi^1 = \frac{(3\theta+2\theta_b)(2\theta+\theta_b)^2 - 2\theta_b - \sqrt{[(3\theta+2\theta_b)(2\theta+\theta_b)^2 - 2\theta_b]^2 + 8\theta^2(2\theta+\theta_b)^4}}{4\theta(2\theta+\theta_b)^2}$$

$$\beta_\pi^2 = \frac{(3\theta+2\theta_b)(2\theta+\theta_b)^2 - 2\theta_b - \sqrt{[(3\theta+2\theta_b)(2\theta+\theta_b)^2 - 2\theta_b]^2 + 8\theta^2(2\theta+\theta_b)^4}}{4\theta(2\theta+\theta_b)^2}$$

β_π^1 is negative. When $\beta > \beta_\pi^2$, $D > 0$. $\hat{\pi}_i$ is greater than π_i

(d) part (d) follows from that:

$$\hat{CS} - CS = (1 - 2\theta - \theta_b)v + \frac{(2\theta+\theta_b)^2 t}{\theta_b} - \frac{2t\beta}{\theta + 3\beta\theta - 2\beta^2\theta + 2\beta\theta_b} - \frac{t}{4} - \frac{(1-\theta_b-\theta)(1-\beta)t}{4} \quad (52)$$

$\hat{CS} - CS < 0$ requires $v < \widetilde{v}$, where

$$\tilde{v} = \frac{t}{1 - 2\theta - \theta_b}\left[\frac{2\beta}{\theta + 3\beta\theta - 2\beta^2\theta + 2\beta\theta_b} + \frac{1}{4} + \frac{(1-\theta_b-\theta)(1-\beta)}{4} - \frac{(2\theta+\theta_b)^2}{\theta_b}\right]$$

(e) part (e) follows from that:

$$\hat{W} - W = (1 - 2\theta - \theta_b)v + \frac{(4\theta+\theta_b)t}{4} - \frac{t}{4} - \frac{(1-\theta_b-\theta)(1-\beta)t}{4} \geq 0 \quad (53)$$

□

A.10 Proof of Lemma 4

Proof. (a) The case without recommender system:

The demand functions can be similarly formulated as in the baseline model:

$$\begin{aligned} D_A &= (\theta + u) + z_0\theta_b \\ D_B &= (\theta - u) + (1 - z_0)\theta_b \end{aligned} \tag{54}$$

The objective function is the same as Equation (3). Each manufacturer's best response to its competitor in stage 1 is characterized by the first-order conditions of Equation (3):

$$\begin{aligned} \tfrac{\partial \pi_A}{\partial p_A} &= (1-\alpha)\left[(\theta+u) + \tfrac{p_B - 2p_A + t}{2t}\theta_b\right] = 0 \\ \tfrac{\partial \pi_B}{\partial p_B} &= (1-\alpha)\left[(\theta-u) + \tfrac{p_A - 2p_B + t}{2t}\theta_b\right] = 0 \end{aligned}$$

Based on these equations, we can derive the manufacturer equilibrium prices as in Lemma 4. Substituting the equilibrium prices into Equations (54) and (3), we can derive the equilibrium demands and manufacturers' profits as in in Lemma 4. Similarly, because $\frac{\pi_R}{\pi_A+\pi_B} = \frac{\alpha}{1-\alpha}$, based on π_A^* and π_B^*, we can derive π_R^*.

Similar to the baseline case, the consumer surplus from consumers who are only aware of product A is $(\theta+u)(v-p_A-\frac{t}{2})$ and from consumers who are only aware of product B is $(\theta-u)(v-p_B-\frac{t}{2})$. Consumer surplus from the fully-informed segment is

$$\theta_b\left[\int_0^{z_0}(v-p_A-zt)dz + \int_{z_0}^1 [v-p_B-(1-z)t]\,dz\right] = \theta_b\left[v - p_A z_0 - p_B(1-z_0) - \tfrac{1}{2}\left(1-2z_0+2z_0^2\right)\right]$$

The consumer surplus from the uninformed segment is zero. Aggregating the consumer surplus from all segments, we can derive the total surplus as in Lemma 4. Social welfare is the sum of consumer surplus, manufacturers' profits, and retailer's profit; that is, $W = CS + \pi_A^* + \pi_B^* + \pi_R^*$, which can be derived as in Lemma 4.[5]

(b) The case with recommender system of $\beta = 1$: In this case, the demand functions are $d_A = z_0$ and$d_B = 1 - z_0$, which are the same as that in the baseline case with $\beta = 1$. Therefore, equilibrium outcome in the case is the special case of Lemma 2 with $\beta = 1$. □

A.11 Proof of Proposition 5

[5]To ensure the market is competitive and fully covered and firms have no profitable deviation, similar to the baseline model, we need to impose some conditions on v and u.

Proof. (a) Notice that

$$\hat{D}_A - D_A = \tfrac{1}{2} - \tfrac{1}{6}(6\theta + 2u + 3\theta_b) \qquad (55)$$
$$\hat{D}_B - D_B = \tfrac{1}{2} - \tfrac{1}{6}(6\theta - 2u + 3\theta_b)$$

Because $(2\theta + \theta_b) \leq 1$, $D_B \leq \hat{D}_B$. According to Equation (55), $D_A \leq \hat{D}_A$ if and only if $u \leq \frac{3}{2}\left[1 - (2\theta + \theta_b)\right]$.

(b) Notice that $p_A = p_B = t$ and $\hat{p}_A \geq \hat{p}_B = \frac{t(6\theta - 2u + 3\theta_b)}{3\theta_b} > \frac{t(3\theta_b)}{3\theta_b} = t$. Therefore, $\hat{p}_i > p_i$.

(c)-(e) Notice that

$$\hat{\pi}_A - \pi_A = \tfrac{(1-\alpha)t}{2}\left[1 - \tfrac{(6\theta+2u+3\theta_b)^2}{9\theta_b}\right]$$
$$\hat{\pi}_B - \pi_B = \tfrac{(1-\alpha)t}{2}\left[1 - \tfrac{(6\theta-2u+3\theta_b)^2}{9\theta_b}\right]$$
$$\hat{\pi}_R - \pi_R = t\alpha\left[1 - \tfrac{4u^2+9(2\theta+\theta_b)^2}{9\theta_b}\right]$$

Therefore, $\hat{\pi}_A \geq \pi_A$ if $u \leq \frac{3}{2}\left[\sqrt{\theta_b} - (2\theta + \theta_b)\right]$. Because $u \geq 0$, the condition can be satisfied only if $\theta \leq \frac{1}{2}(\sqrt{\theta_b} - \theta_b)$. Similarly, $\hat{\pi}_B \leq \pi_B$ if $u \leq \frac{3}{2}\left[(2\theta + \theta_b) - \sqrt{\theta_b}\right]$, and the condition can be satisfied only if $\theta \geq \frac{1}{2}(\sqrt{\theta_b} - \theta_b)$. $\hat{\pi}_R \geq \pi_R$ if $u \leq \frac{3}{2}\left[\sqrt{\theta_b - (2\theta + \theta_b)^2}\right]$, and the condition can be satisfied only if $\theta \leq \frac{1}{2}(\sqrt{\theta_b} - \theta_b)$.

(d) Notice that

$$\hat{CS} - CS = \left(v - \tfrac{5t}{4}\right) - \left[(2\theta + \theta_b)\left(v - \tfrac{5t}{4}\right) - \tfrac{5t}{4}\left(2\theta + \tfrac{16(2u^2+9\theta^2)}{45\theta_b}\right)\right] \geq 0$$

where the last inequality is because $(2\theta + \theta_b) \leq 1$.

(e) Notice that

$$\hat{W} - W = \left(v - \tfrac{t}{4}\right) - \left[(2\theta + \theta_b)\left(v - \tfrac{t}{4}\right) - \tfrac{t}{4}\left(2\theta + \tfrac{16u^2}{9\theta_b}\right)\right] \geq 0$$

where the last inequality is because $(2\theta + \theta_b) \leq 1$. □